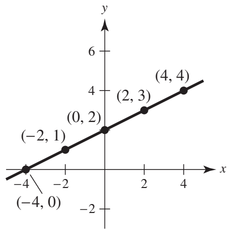
Answers to Odd-Numbered Exercises

Chapter P

Section P.1 *(page 8)*

1. b **2.** d **3.** a **4.** c

5.



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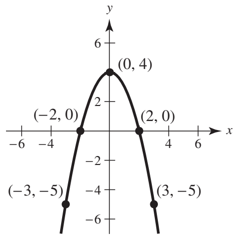

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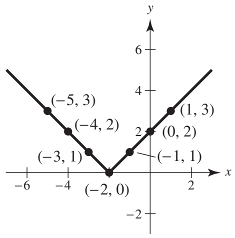

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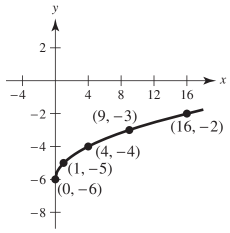

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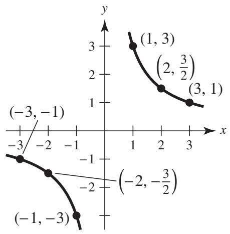

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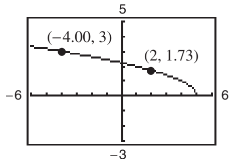


(a) $y \approx 1.73$ (b) $x = -4$

17. $(0, -5), \left(\frac{5}{2}, 0\right)$ **19.** $(0, -2), (-2, 0), (1, 0)$

21. $(0, 0), (4, 0), (-4, 0)$ **23.** $(0, 2), (4, 0)$ **25.** $(0, 0)$

27. Symmetric with respect to the y-axis

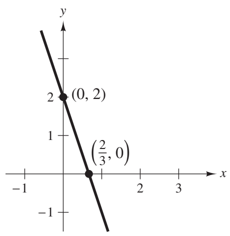
29. Symmetric with respect to the x-axis

31. Symmetric with respect to the origin **33.** No symmetry

35. Symmetric with respect to the origin

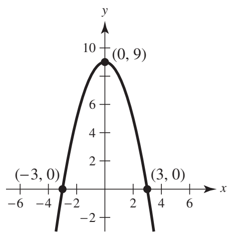
37. Symmetric with respect to the y-axis

39.



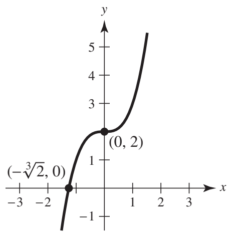
Symmetry: none

41.



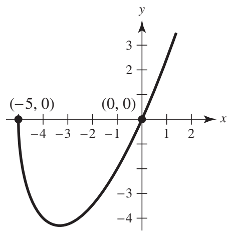
Symmetry: y-axis

43.



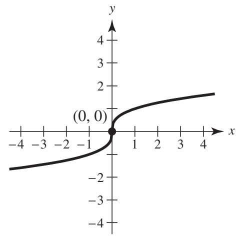
Symmetry: none

45.



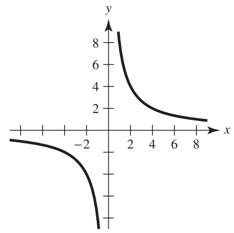
Symmetry: none

47.



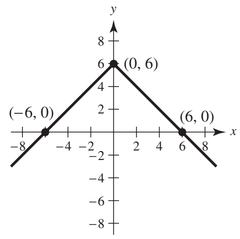
Symmetry: origin

49.



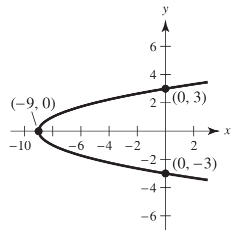
Symmetry: origin

51.



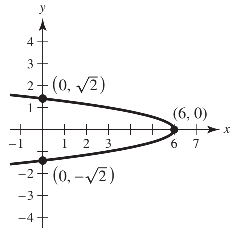
Symmetry: y-axis

53.



Symmetry: x-axis

55.



Symmetry: x-axis

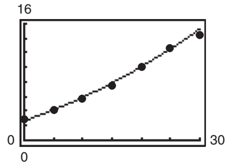
57. $(3, 5)$

59. $(-1, 5), (2, 2)$ **61.** $(-1, -2), (2, 1)$

63. $(-1, -5), (0, -1), (2, 1)$ **65.** $(-2, 2), \left(-3, \sqrt{3}\right)$

67. (a) $y = 0.005t^2 + 0.27t + 2.7$

(b)



The model is a good fit for the data.

(c) $21.5 trillion

69. 4480 units

71. (a) $k = 4$ (b) $k = -\frac{1}{8}$

(c) All real numbers k (d) $k = 1$

73. Answers will vary. Sample answer: $y = (x + 4)(x - 3)(x - 8)$

75. (a) Proof (b) Proof

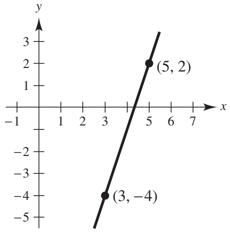
77. False. $(4, -5)$ is not a point on the graph of $x = y^2 - 29$.

79. True

Section P.2 *(page 16)*

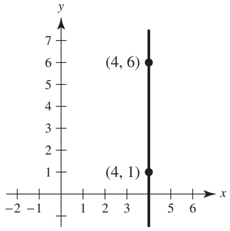
1. $m = 2$ **3.** $m = -1$

5.



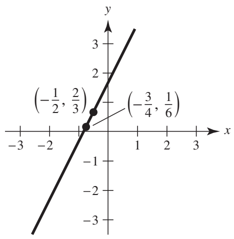
$m = 3$

7.



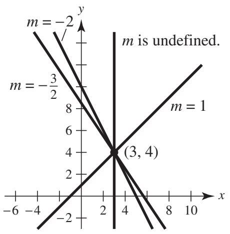
m is undefined.

9.



$m = 2$

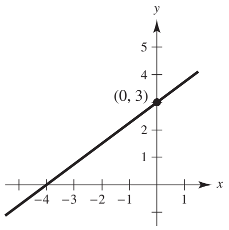
11.



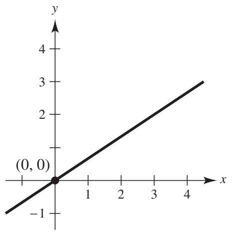
13. Answers will vary. Sample answers: (0, 2), (1, 2), (5, 2)

15. Answers will vary. Sample answers: (0, 10), (2, 4), (3, 1)

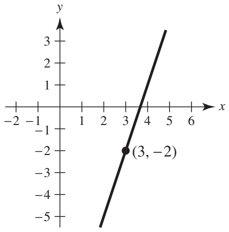
17. $3x - 4y + 12 = 0$



19. $2x - 3y = 0$



21. $3x - y - 11 = 0$

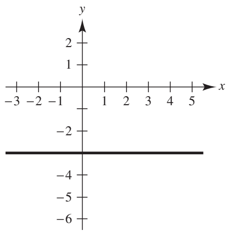


23. (a) $\frac{1}{3}$ (b) $10\sqrt{10}$ ft

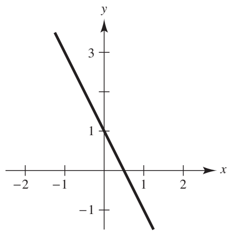
25. $m = 4, (0, -3)$ **27.** $m = -\frac{1}{5}, (0, 4)$

29. m is undefined, no y-intercept

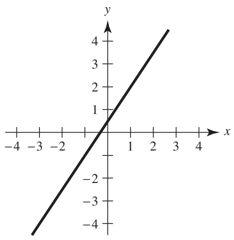
31.



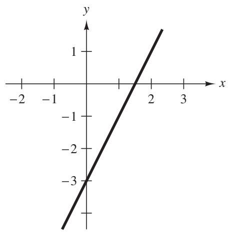
33.



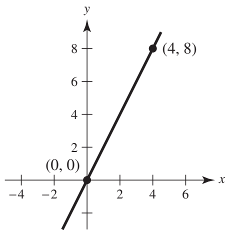
35.



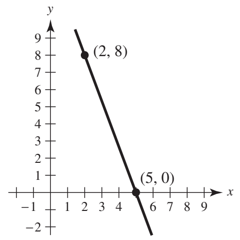
37.



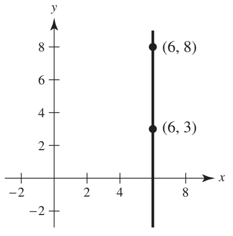
39. $2x - y = 0$



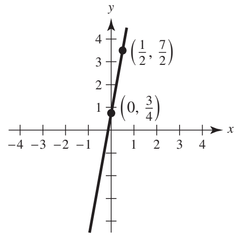
41. $8x + 3y - 40 = 0$



43. $x - 6 = 0$



45. $22x - 4y + 3 = 0$



47. $x - 3 = 0$ **49.** $3x + 2y - 6 = 0$ **51.** $x + y - 3 = 0$

53. $x + 2y - 5 = 0$ **55.** (a) $x + 7 = 0$ (b) $y + 2 = 0$

57. (a) $x - y + 3 = 0$ (b) $x + y - 7 = 0$

59. (a) $2x - y - 3 = 0$ (b) $x + 2y - 4 = 0$

61. (a) $40x - 24y - 9 = 0$ (b) $24x + 40y - 53 = 0$

63. $V = 250t + 1350$ **65.** $V = -1600t + 20{,}400$

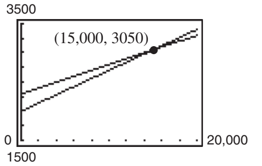
67. Not collinear, because $m_1 \neq m_2$

69. $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$ **71.** $\left(b, \dfrac{a^2 - b^2}{c}\right)$

73. (a) The line is parallel to the x-axis when $a = 0$ and $b \neq 0$.
(b) The line is parallel to the y-axis when $b = 0$ and $a \neq 0$.
(c) Answers will vary. Sample answer: $a = -5$ and $b = 8$
(d) Answers will vary. Sample answer: $a = 5$ and $b = 2$
(e) $a = \frac{5}{2}$ and $b = 3$

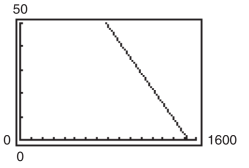
75. $5F - 9C - 160 = 0$; 72°F ≈ 22.2°C

77. (a) Current job: $W = 2000 + 0.07s$
Job offer: $W = 2300 + 0.05s$
(b)



You will make more money at the job offer until you sell \$15,000. When your sales exceed \$15,000, your current job will pay you more.
(c) No, because you will make more money at your current job.

79. (a) $x = (1530 - p)/15$
(b)



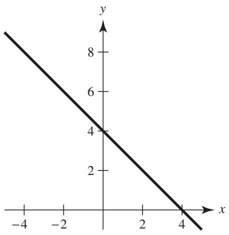
45 units
(c) 49 units

81. $12y + 5x - 169 = 0$ **83.** $(5\sqrt{2})/2$ **85.** $2\sqrt{2}$

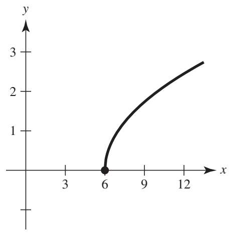
87–91. Proofs **93.** True **95.** True

Section P.3 *(page 27)*

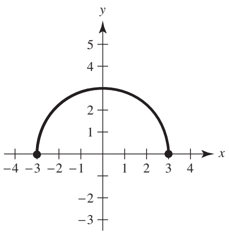
1. (a) -4 (b) -25 (c) $7b-4$ (d) $7x-11$
3. (a) 5 (b) 0 (c) 1 (d) $4+2t-t^2$
5. (a) 1 (b) 0 (c) $-\frac{1}{2}$ (d) 1
7. $3x^2+3x\,\Delta x+(\Delta x)^2,\ \Delta x \neq 0$
9. $\left(\sqrt{x-1}-x+1\right)/[(x-2)(x-1)]$
11. Domain: $(-\infty, \infty)$; Range: $[0, \infty)$
13. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$
15. Domain: $[0, \infty)$; Range: $[0, \infty)$
17. Domain: $[-4, 4]$; Range: $[0, 4]$
19. Domain: All real numbers t such that $t \neq 4n+2$, where n is an integer; Range: $(-\infty, -1] \cup [1, \infty)$
21. Domain: $(-\infty, 0) \cup (0, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$
23. Domain: $[0, 1]$
25. Domain: All real numbers x such that $x \neq 2n\pi$, where n is an integer
27. Domain: $(-\infty, -3) \cup (-3, \infty)$
29. (a) -1 (b) 2 (c) 6 (d) $2t^2+4$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 1) \cup [2, \infty)$
31. (a) 4 (b) 0 (c) -2 (d) $-b^2$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 0] \cup [1, \infty)$

33.


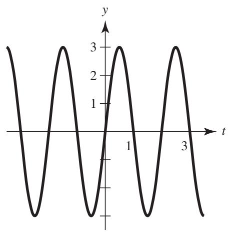
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

35.


Domain: $[6, \infty)$
Range: $[0, \infty)$

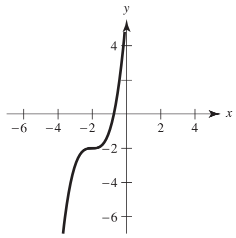
37.


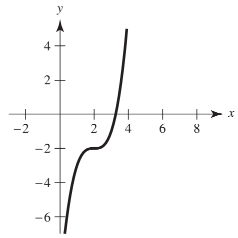
Domain: $[-3, 3]$
Range: $[0, 3]$

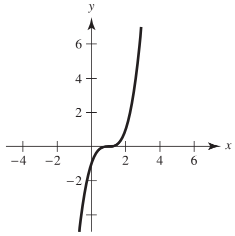
39.


Domain: $(-\infty, \infty)$
Range: $[-3, 3]$

41. The student travels $\frac{1}{2}$ mile/minute during the first 4 minutes, is stationary for the next 2 minutes, and travels 1 mile/minute during the final 4 minutes.
43. y is not a function of x. **45.** y is a function of x.
47. y is not a function of x. **49.** y is not a function of x.
51. Horizontal shift to the right two units
$y=\sqrt{x-2}$
53. Horizontal shift to the right two units and vertical shift down one unit
$y=(x-2)^2-1$
55. *d* **56.** *b* **57.** *c* **58.** *a* **59.** *e* **60.** *g*

61. (a)


(b)


(c)


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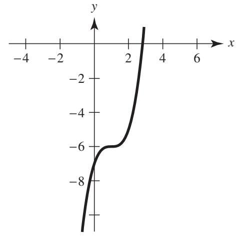
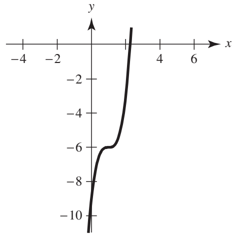
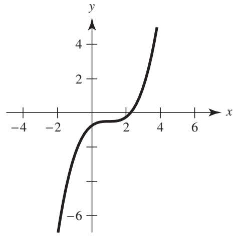
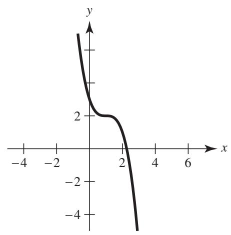
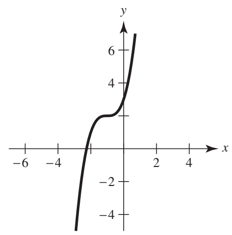


(e)


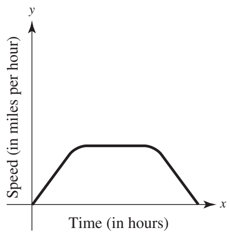
(f)


(g)


(h)


63. (a) $3x$ (b) $3x-8$ (c) $12x-16$ (d) $\frac{3}{4}x-1$
65. (a) 0 (b) 0 (c) -1 (d) $\sqrt{15}$
(e) $\sqrt{x^2-1}$ (f) $x-1\ (x \geq 0)$
67. $(f \circ g)(x)=x$; Domain: $[0, \infty)$
$(g \circ f)(x)=|x|$; Domain: $(-\infty, \infty)$
No, their domains are different.
69. $(f \circ g)(x)=3/(x^2-1)$;
Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
$(g \circ f)(x)=\left(9/x^2\right)-1$; Domain: $(-\infty, 0) \cup (0, \infty)$
No
71. (a) 4 (b) -2
(c) Undefined. The graph of g does not exist at $x=-5$.
(d) 3 (e) 2
(f) Undefined. The graph of f does not exist at $x=-4$.
73. Answers will vary.
Sample answer: $f(x)=\sqrt{x}$; $g(x)=x-2$; $h(x)=2x$
75. (a) $\left(\frac{3}{2}, 4\right)$ (b) $\left(\frac{3}{2}, -4\right)$
77. f is even. g is neither even nor odd. h is odd.
79. Even; zeros: $x=-2, 0, 2$
81. Odd; zeros: $x=0, \dfrac{\pi}{2}+n\pi$, where n is an integer
83. $f(x)=-5x-6, -2 \leq x \leq 0$ **85.** $y=-\sqrt{-x}$

87. Answers will vary.
Sample answer:

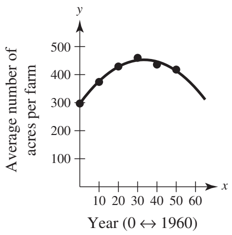


89. Answers will vary.
Sample answer:

91. $c = 25$

93. (a) $T(4) = 16°C$, $T(15) \approx 23°C$
(b) The changes in temperature occur 1 hour later.
(c) The temperatures are 1° lower.

95. (a)



(b) $A(25) \approx$ 443 acres/farm

97. $f(x) = |x| + |x - 2| = \begin{cases} 2x - 2, & x \ge 2 \\ 2, & 0 < x < 2 \\ -2x + 2, & x \le 0 \end{cases}$

99–101. Proofs **103.** $L = \sqrt{x^2 + \left(\frac{2x}{x - 3}\right)^2}$

105. False. For example, if $f(x) = x^2$, then $f(-1) = f(1)$.

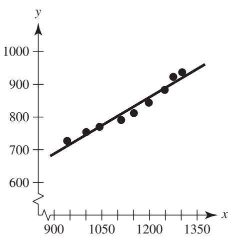
107. True

109. False. $f(x) = 0$ is symmetric with respect to the x-axis.

111. Putnam Problem A1, 1988

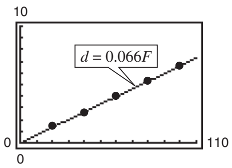
Section P.4 *(page 34)*

1. (a) and (b)



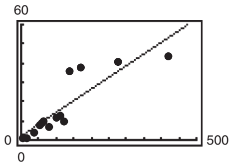
(c) $790

3. (a) $d = 0.066F$
(b)



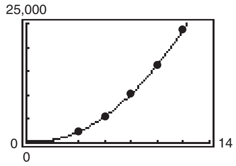
The model fits well.
(c) 3.63 cm

5. (a) $y = 0.122x + 2.07$, $r \approx 0.87$
(b)



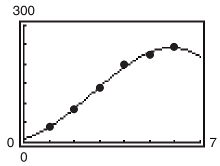
(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national product of the country. The three countries that differ most from the linear model are Canada, Italy, and Japan.
(d) $y = 0.142x - 1.66$, $r \approx 0.97$

7. (a) $S = 180.89x^2 - 205.79x + 272$
(b)



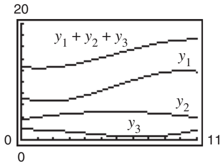
(c) When $x = 2$, $S \approx 583.98$ pounds.
(d) About 4 times greater
(e) About 4.37 times greater; No; Answers will vary.

9. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$
(b)



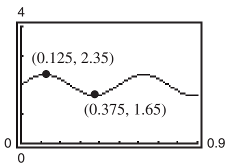
(c) 214 hp

11. (a) $y_1 = -0.0172t^3 + 0.305t^2 - 0.87t + 7.3$
$y_2 = -0.038t^2 + 0.45t + 3.5$
$y_3 = 0.0063t^3 - 0.072t^2 + 0.02t + 1.8$
(b)



About 15.31 cents/mi

13. (a) Yes. At time t, there is one and only one displacement y.
(b) Amplitude: 0.35; Period: 0.5
(c) $y = 0.35 \sin(4\pi t) + 2$
(d)



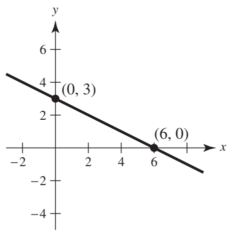
The model appears to fit the data well.

15. Answers will vary. **17.** Putnam Problem A2, 2004

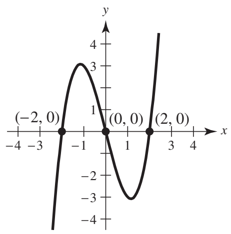
Review Exercises for Chapter P *(page 37)*

1. $\left(\frac{8}{5}, 0\right), (0, -8)$ **3.** $(3, 0), \left(0, \frac{3}{4}\right)$ **5.** Not symmetric

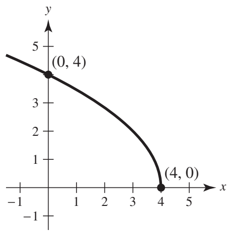
7. Symmetric with respect to the x-axis, the y-axis, and the origin

9. 

Symmetry: none

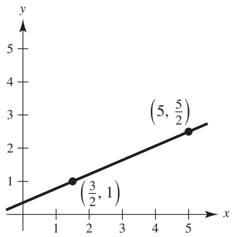
11. 

Symmetry: origin

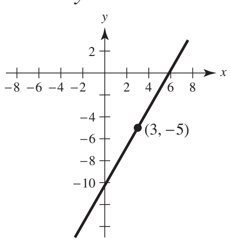
13. 

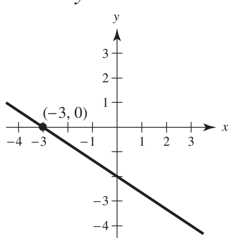
Symmetry: none

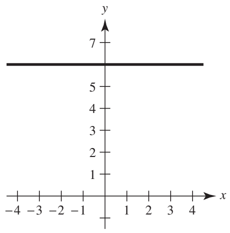
15. $(-2, 3)$ **17.** $(-2, 3), (3, 8)$

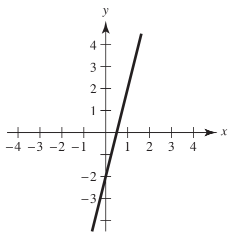
19. 

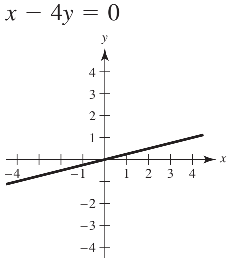
$m = \frac{3}{7}$

21. $7x - 4y - 41 = 0$


23. $2x + 3y + 6 = 0$


25. 

27. 

29. $x - 4y = 0$


31. (a) $7x - 16y + 101 = 0$
(b) $5x - 3y + 30 = 0$
(c) $4x - 3y + 27 = 0$
(d) $x + 3 = 0$

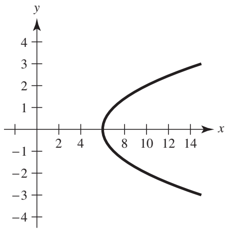
33. $V = 12{,}500 - 850t$; \$9950

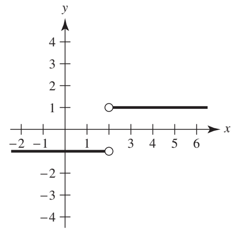
35. (a) 4 (b) 29 (c) -11 (d) $5t + 9$

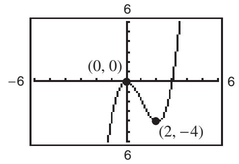
37. $8x + 4\,\Delta x, \Delta x \neq 0$

39. Domain: $(-\infty, \infty)$; Range: $[3, \infty)$

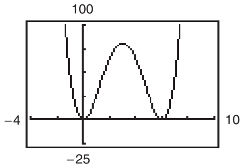
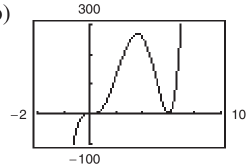
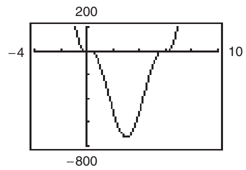
41. Domain: $(-\infty, \infty)$; Range: $(-\infty, 0]$

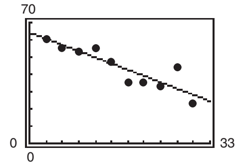
43. 
Not a function

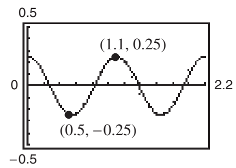
45. 
Function

47. $f(x) = x^3 - 3x^2$


(a) $g(x) = -x^3 + 3x^2 + 1$
(b) $g(x) = (x - 2)^3 - 3(x - 2)^2 + 1$

49. (a) 
(b) 
(c) 

51. (a) $y = -1.204x + 64.2667$
(b) 
(c) The data point $(27, 44)$ is probably an error. Without this point, the new model is $y = -1.4344x + 66.4387$.

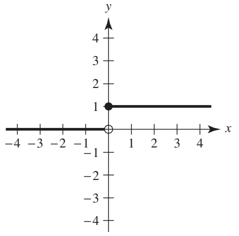
53. (a) Yes. For each time t, there corresponds one and only one displacement y.
(b) Amplitude: 0.25; Period: 1.1 (c) $y \approx \frac{1}{4}\cos(5.7t)$
(d) 

The model appears to fit the data.

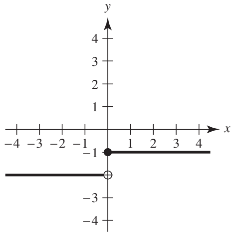
P.S. Problem Solving *(page 39)*

1. (a) Center: $(3, 4)$; Radius: 5
(b) $y = -\frac{3}{4}x$ (c) $y = \frac{3}{4}x - \frac{9}{2}$ (d) $\left(3, -\frac{9}{4}\right)$

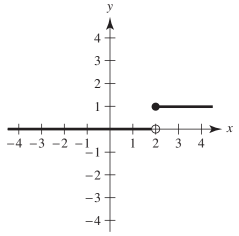
3.



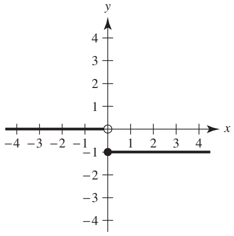
(a)



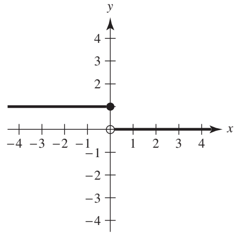
(b)



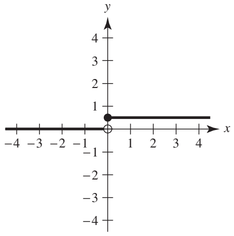
(c)



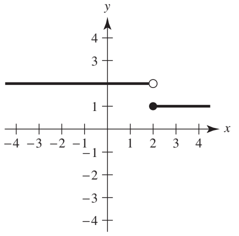
(d)



(e)

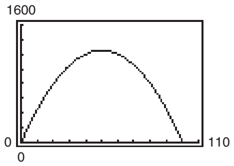


(f)



5. (a) $A(x) = x[(100 - x)/2]$; Domain: $(0, 100)$

(b)



Dimensions $50 \text{ m} \times 25 \text{ m}$ yield maximum area of 1250 m^2.

(c) $50 \text{ m} \times 25 \text{ m}$; Area $= 1250 \text{ m}^2$

7. $T(x) = \left[2\sqrt{4 + x^2} + \sqrt{(3 - x)^2 + 1}\right]/4$

9. (a) 5, less (b) 3, greater (c) 4.1, less (d) $4 + h$ (e) 4; Answers will vary.

11. (a) Domain: $(-\infty, 1) \cup (1, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$

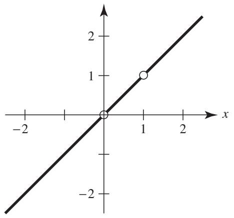
(b) $f(f(x)) = \dfrac{x - 1}{x}$

Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = x$

Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

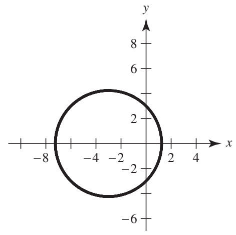
(d)



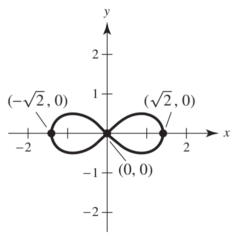
The graph is not a line because there are holes at $x = 0$ and $x = 1$.

13. (a) $x \approx 1.2426, -7.2426$

(b) $(x + 3)^2 + y^2 = 18$



15. Proof



Chapter 1

Section 1.1 *(page 47)*

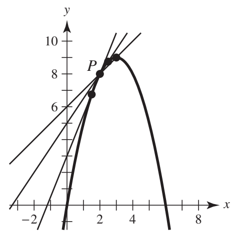
1. Precalculus: 300 ft

3. Calculus: Slope of the tangent line at $x = 2$ is 0.16.

5. (a) Precalculus: 10 square units

(b) Calculus: 5 square units

7. (a)



(b) $1; \frac{3}{2}; \frac{5}{2}$

(c) 2. Use points closer to P.

9. Area ≈ 10.417; Area ≈ 9.145; Use more rectangles.

Section 1.2 *(page 55)*

1.

x	3.9	3.99	3.999	4
$f(x)$	0.2041	0.2004	0.2000	?

x	4.001	4.01	4.1
$f(x)$	0.2000	0.1996	0.1961

$\lim_{x \to 4} \dfrac{x - 4}{x^2 - 3x - 4} \approx 0.2000 \left(\text{Actual limit is } \dfrac{1}{5}.\right)$

3.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.5132	0.5013	0.5001	?

x	0.001	0.01	0.1
$f(x)$	0.4999	0.4988	0.4881

$\lim_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x} \approx 0.5000 \left(\text{Actual limit is } \dfrac{1}{2}.\right)$

5.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9983	0.99998	1.0000	?

x	0.001	0.01	0.1
$f(x)$	1.0000	0.99998	0.9983

$\lim_{x\to 0} \frac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.)

7.

x	0.9	0.99	0.999	1
$f(x)$	0.2564	0.2506	0.2501	?

x	1.001	1.01	1.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to 1} \frac{x-2}{x^2+x-6} \approx 0.2500 \left(\text{Actual limit is } \frac{1}{4}.\right)$

9.

x	0.9	0.99	0.999	1
$f(x)$	0.7340	0.6733	0.6673	?

x	1.001	1.01	1.1
$f(x)$	0.6660	0.6600	0.6015

$\lim_{x\to 1} \frac{x^4-1}{x^6-1} \approx 0.6666 \left(\text{Actual limit is } \frac{2}{3}.\right)$

11.

x	-6.1	-6.01	-6.001	-6
$f(x)$	-0.1248	-0.1250	-0.1250	?

x	-5.999	-5.99	-5.9
$f(x)$	-0.1250	-0.1250	-0.1252

$\lim_{x\to -6} \frac{\sqrt{10-x}-4}{x+6} \approx -0.1250 \left(\text{Actual limit is } -\frac{1}{8}.\right)$

13.

x	-0.1	-0.01	-0.001	0
$f(x)$	1.9867	1.9999	2.0000	?

x	0.001	0.01	0.1
$f(x)$	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.)

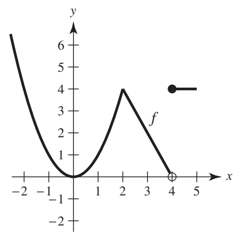
15. 1 **17.** 2

19. Limit does not exist. The function approaches 1 from the right side of 2, but it approaches -1 from the left side of 2.

21. Limit does not exist. The function oscillates between 1 and -1 as x approaches 0.

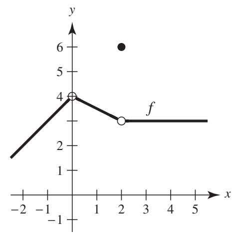
23. (a) 2
(b) Limit does not exist. The function approaches 1 from the right side of 1, but it approaches 3.5 from the left side of 1.
(c) Value does not exist. The function is undefined at $x = 4$.
(d) 2

25.



$\lim_{x\to c} f(x)$ exists for all points on the graph except where $c = 4$.

27.



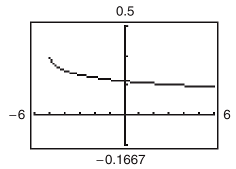
29. $\delta = 0.4$ **31.** $\delta = \frac{1}{11} \approx 0.091$

33. $L = 8$. Let $\delta = 0.01/3 \approx 0.0033$.

35. $L = 1$. Let $\delta = 0.01/5 = 0.002$. **37.** 6 **39.** -3

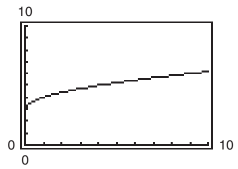
41. 3 **43.** 0 **45.** 10 **47.** 2 **49.** 4

51.



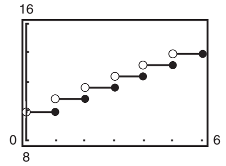
$\lim_{x\to 4} f(x) = \frac{1}{6}$
Domain: $[-5, 4) \cup (4, \infty)$
The graph has a hole at $x = 4$.

53.



$\lim_{x\to 9} f(x) = 6$
Domain: $[0, 9) \cup (9, \infty)$
The graph has a hole at $x = 9$.

55. (a)



(b)

t	3	3.3	3.4	3.5
C	11.57	12.36	12.36	12.36

t	3.6	3.7	4
C	12.36	12.36	12.36

$\lim_{t\to 3.5} C(t) = 12.36$

(c)

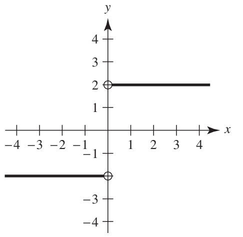
t	2	2.5	2.9	3
C	10.78	11.57	11.57	11.57

t	3.1	3.5	4
C	12.36	12.36	12.36

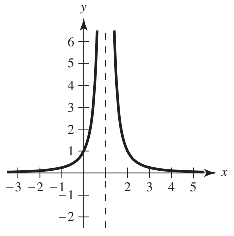
The limit does not exist because the limits from the right and left are not equal.

57. Answers will vary. Sample answer: As x approaches 8 from either side, $f(x)$ becomes arbitrarily close to 25.

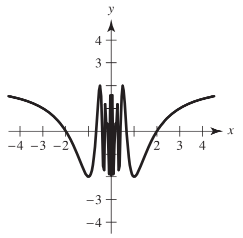
59. (i) The values of f approach different numbers as x approaches c from different sides of c.



(ii) The values of f increase or decrease without bound as x approaches c.



(iii) The values of f oscillate between two fixed numbers as x approaches c.



61. (a) $r = \dfrac{3}{\pi} \approx 0.9549$ cm

(b) $\dfrac{5.5}{2\pi} \le r \le \dfrac{6.5}{2\pi}$, or approximately $0.8754 < r < 1.0345$

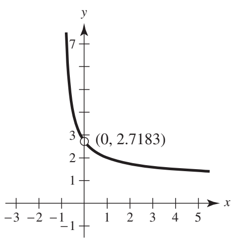
(c) $\lim_{r\to 3/\pi} 2\pi r = 6$; $\varepsilon = 0.5$; $\delta \approx 0.0796$

63.

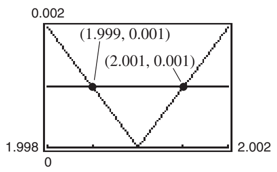
x	-0.001	-0.0001	-0.00001
$f(x)$	2.7196	2.7184	2.7183

x	0.00001	0.0001	0.001
$f(x)$	2.7183	2.7181	2.7169

$\lim_{x\to 0} f(x) \approx 2.7183$



65.



$\delta = 0.001$, $(1.999, 2.001)$

67. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

69. False. See Exercise 17.

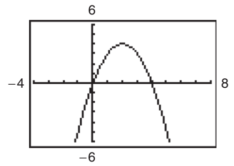
71. Yes. As x approaches 0.25 from either side, $\sqrt{x}$ becomes arbitrarily close to 0.5.

73. $\lim_{x\to 0} \dfrac{\sin nx}{x} = n$ **75–77.** Proofs

79. Putnam Problem B1, 1986

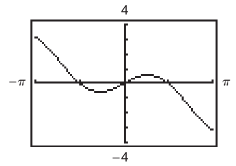
Section 1.3 *(page 67)*

1.



(a) 0 (b) -5

3.



(a) 0 (b) About 0.52 or $\pi/6$

5. 8 **7.** -1 **9.** 0 **11.** 7 **13.** 2 **15.** 1
17. $1/2$ **19.** $1/5$ **21.** 7 **23.** (a) 4 (b) 64 (c) 64
25. (a) 3 (b) 2 (c) 2 **27.** 1 **29.** $1/2$ **31.** 1
33. $1/2$ **35.** -1 **37.** (a) 10 (b) 5 (c) 6 (d) $3/2$
39. (a) 64 (b) 2 (c) 12 (d) 8

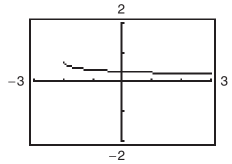
41. $f(x) = \dfrac{x^2 + 3x}{x}$ and $g(x) = x + 3$ agree except at $x = 0$.
$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 3$

43. $f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.
$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = -2$

45. $f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.
$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = 12$

47. -1 **49.** $1/8$ **51.** $5/6$ **53.** $1/6$ **55.** $\sqrt{5}/10$
57. $-1/9$ **59.** 2 **61.** $2x - 2$ **63.** $1/5$ **65.** 0
67. 0 **69.** 0 **71.** 1 **73.** $3/2$

75.



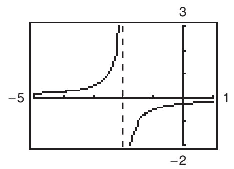
The graph has a hole at $x = 0$.

Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	0.354	0.353	0.349

$\lim_{x\to 0} \dfrac{\sqrt{x+2} - \sqrt{2}}{x} \approx 0.354$; Actual limit is $\dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$.

77.



The graph has a hole at $x = 0$.

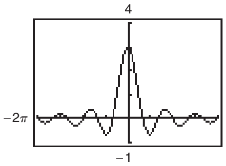
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001
$f(x)$	-0.263	-0.251	-0.250

x	0.001	0.01	0.1
$f(x)$	-0.250	-0.249	-0.238

$\lim_{x\to 0} \dfrac{[1/(2 + x)] - (1/2)}{x} \approx -0.250$; Actual limit is $-\dfrac{1}{4}$.

79.



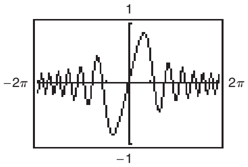
The graph has a hole at $t = 0$.

Answers will vary. Sample answer:

t	-0.1	-0.01	0	0.01	0.1
$f(t)$	2.96	2.9996	?	2.9996	2.96

$\lim_{t\to 0} \frac{\sin 3t}{t} \approx 3.0000$; Actual limit is 3.

81.



The graph has a hole at $x = 0$.

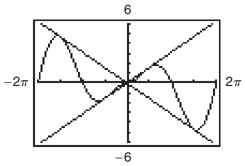
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-0.1	-0.01	-0.001	?	0.001	0.01	0.1

$\lim_{x\to 0} \frac{\sin x^2}{x} = 0$; Actual limit is 0.

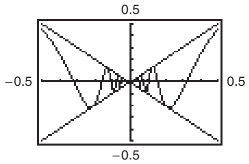
83. 3 **85.** $2x - 4$ **87.** $-1/(x + 3)^2$ **89.** 4

91.



0

93.



0

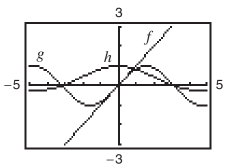
The graph has a hole at $x = 0$.

95. (a) f and g agree at all but one point if c is a real number such that $f(x) = g(x)$ for all $x \neq c$.

(b) Sample answer: $f(x) = \frac{x^2 - 1}{x - 1}$ and $g(x) = x + 1$ agree at all points except $x = 1$.

97. If a function f is squeezed between two functions h and g, $h(x) \leq f(x) \leq g(x)$, and h and g have the same limit L as $x \to c$, then $\lim_{x\to c} f(x)$ exists and equals L.

99.



The magnitudes of $f(x)$ and $g(x)$ are approximately equal when x is close to 0. Therefore, their ratio is approximately 1.

101. -64 ft/sec (speed $= 64$ ft/sec) **103.** -29.4 m/sec

105. Let $f(x) = 1/x$ and $g(x) = -1/x$.

$\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$$\lim_{x\to 0} [f(x) + g(x)] = \lim_{x\to 0} \left[\frac{1}{x} + \left(-\frac{1}{x}\right)\right] = \lim_{x\to 0} 0 = 0$$

and therefore does exist.

107–111. Proofs

113. Let $f(x) = \begin{cases} 4, & x \geq 0 \\ -4, & x < 0 \end{cases}$.

$\lim_{x\to 0} |f(x)| = \lim_{x\to 0} 4 = 4$

$\lim_{x\to 0} f(x)$ does not exist because for $x < 0$, $f(x) = -4$ and for $x \geq 0$, $f(x) = 4$.

115. False. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0.

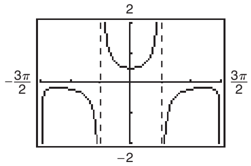
117. True.

119. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.

121. Proof

123. (a) All $x \neq 0, \frac{\pi}{2} + n\pi$

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent from the graph.

(c) $\frac{1}{2}$ (d) $\frac{1}{2}$

Section 1.4 *(page 79)*

1. (a) 3 (b) 3 (c) 3; $f(x)$ is continuous on $(-\infty, \infty)$.

3. (a) 0 (b) 0 (c) 0; Discontinuity at $x = 3$

5. (a) -3 (b) 3 (c) Limit does not exist. Discontinuity at $x = 2$

7. $\frac{1}{16}$ **9.** $\frac{1}{10}$

11. Limit does not exist. The function decreases without bound as x approaches -3 from the left.

13. -1 **15.** $-1/x^2$ **17.** $5/2$ **19.** 2

21. Limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

23. 8

25. Limit does not exist. The function approaches 5 from the left side of 3 but approaches 6 from the right side of 3.

27. Discontinuities at $x = -2$ and $x = 2$

29. Discontinuities at every integer

31. Continuous on $[-7, 7]$ **33.** Continuous on $[-1, 4]$

35. Nonremovable discontinuity at $x = 0$

37. Continuous for all real x

39. Nonremovable discontinuities at $x = -2$ and $x = 2$

41. Continuous for all real x

43. Nonremovable discontinuity at $x = 1$
Removable discontinuity at $x = 0$

45. Continuous for all real x

47. Removable discontinuity at $x = -2$
Nonremovable discontinuity at $x = 5$

49. Nonremovable discontinuity at $x = -7$

51. Continuous for all real x

53. Nonremovable discontinuity at $x = 2$

55. Continuous for all real x

57. Nonremovable discontinuities at integer multiples of $\pi/2$

59. Nonremovable discontinuities at each integer

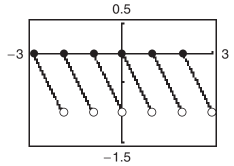
61. $a = 7$ **63.** $a = 2$ **65.** $a = -1,\ b = 1$

67. Continuous for all real x

69. Nonremovable discontinuities at $x = 1$ and $x = -1$

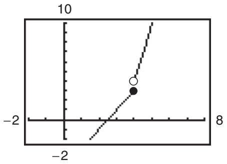
71. Continuous on the open intervals $\ldots, (-3\pi, -\pi), (-\pi, \pi), (\pi, 3\pi), \ldots$

73.



Nonremovable discontinuity at each integer

75.



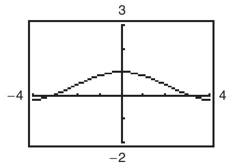
Nonremovable discontinuity at $x = 4$

77. Continuous on $(-\infty, \infty)$ **79.** Continuous on $[0, \infty)$

81. Continuous on the open intervals $\ldots, (-6, -2), (-2, 2), (2, 6), \ldots$

83. Continuous on $(-\infty, \infty)$

85.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

87. Because $f(x)$ is continuous on the interval $[1, 2]$ and $f(1) = 37/12$ and $f(2) = -8/3$, by the Intermediate Value Theorem there exists a real number c in $[1, 2]$ such that $f(c) = 0$.

89. Because $f(x)$ is continuous on the interval $[0, \pi]$ and $f(0) = -3$ and $f(\pi) \approx 8.87$, by the Intermediate Value Theorem there exists a real number c in $[0, \pi]$ such that $f(c) = 0$.

91. 0.68, 0.6823 **93.** 0.56, 0.5636

95. $f(3) = 11$ **97.** $f(2) = 4$

99. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

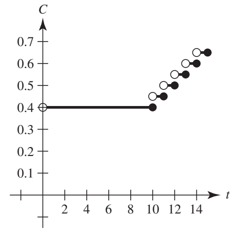
101. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

103. True

105. False. A rational function can be written as $P(x)/Q(x)$, where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

107. The functions differ by 1 for non-integer values of x.

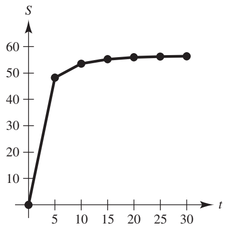
109. $C = \begin{cases} 0.40, & 0 < t \le 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10,\ t \text{ is not an integer} \\ 0.40 + 0.05(t - 10), & t > 10,\ t \text{ is an integer} \end{cases}$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

111–113. Proofs **115.** Answers will vary.

117. (a)

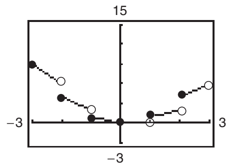


(b) There appears to be a limiting speed, and a possible cause is air resistance.

119. $c = \left(-1 \pm \sqrt{5}\right)/2$

121. Domain: $[-c^2, 0) \cup (0, \infty)$; Let $f(0) = 1/(2c)$

123. $h(x)$ has a nonremovable discontinuity at every integer except 0.



125. Putnam Problem B2, 1988

Section 1.5 *(page 88)*

1. $\displaystyle\lim_{x\to -2^+} 2\left|\frac{x}{x^2 - 4}\right| = \infty, \quad \lim_{x\to -2^-} 2\left|\frac{x}{x^2 - 4}\right| = \infty$

3. $\displaystyle\lim_{x\to -2^+} \tan(\pi x/4) = -\infty, \quad \lim_{x\to -2^-} \tan(\pi x/4) = \infty$

5. $\displaystyle\lim_{x\to 4^+} \frac{1}{x - 4} = \infty, \quad \lim_{x\to 4^-} \frac{1}{x - 4} = -\infty$

7. $\displaystyle\lim_{x\to 4^+} \frac{1}{(x - 4)^2} = \infty, \quad \lim_{x\to 4^-} \frac{1}{(x - 4)^2} = \infty$

9.

x	-3.5	-3.1	-3.01	-3.001	-3
$f(x)$	0.31	1.64	16.6	167	?

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-167	-16.7	-1.69	-0.36

$\displaystyle\lim_{x\to -3^+} f(x) = -\infty; \quad \lim_{x\to -3^-} f(x) = \infty$

11.

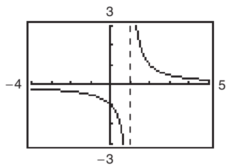
x	-3.5	-3.1	-3.01	-3.001	-3
$f(x)$	3.8	16	151	1501	?

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-1499	-149	-14	-2.3

$\lim_{x\to -3^+} f(x) = -\infty;\ \lim_{x\to -3^-} f(x) = \infty$

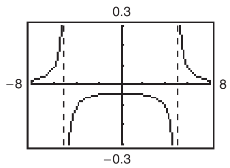
13. $x = 0$ **15.** $x = \pm 2$ **17.** No vertical asymptote
19. $x = -2, x = 1$ **21.** $x = 0, x = 3$
23. No vertical asymptote **25.** $x = n$, n is an integer.
27. $t = n\pi$, n is a nonzero integer.
29. Removable discontinuity at $x = -1$
31. Vertical asymptote at $x = -1$
33. ∞ **35.** ∞ **37.** $-\frac{1}{5}$ **39.** $-\infty$ **41.** $-\infty$
43. ∞ **45.** 0 **47.** ∞

49.



$\lim_{x\to 1^+} f(x) = \infty$

51.

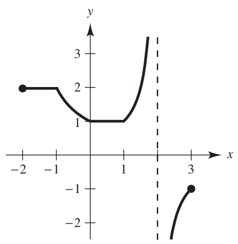


$\lim_{x\to 5^-} f(x) = -\infty$

53. Answers will vary.

55. Answers will vary. Sample answer: $f(x) = \dfrac{x-3}{x^2-4x-12}$

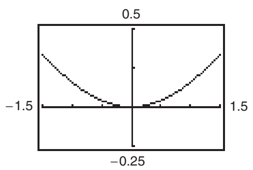
57.



59. (a)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0411	0.0067	0.0017

x	0.01	0.001	0.0001
$f(x)$	≈ 0	≈ 0	≈ 0

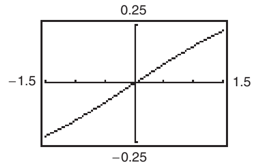


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x} = 0$

(b)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0823	0.0333	0.0167

x	0.01	0.001	0.0001
$f(x)$	0.0017	≈ 0	≈ 0

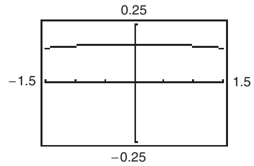


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^2} = 0$

(c)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.1646	0.1663	0.1666

x	0.01	0.001	0.0001
$f(x)$	0.1667	0.1667	0.1667

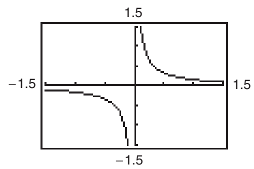


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^3} = 0.1667\ (1/6)$

(d)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.3292	0.8317	1.6658

x	0.01	0.001	0.0001
$f(x)$	16.67	166.7	1667.0



$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^4} = \infty$

For $n > 3$, $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^n} = \infty$.

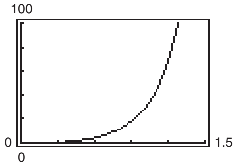
61. (a) $\dfrac{7}{12}$ ft/sec (b) $\dfrac{3}{2}$ ft/sec

(c) $\lim_{x\to 25^-} \dfrac{2x}{\sqrt{625 - x^2}} = \infty$

63. (a) $A = 50 \tan \theta - 50\theta$; Domain: $(0, \pi/2)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta \to \pi/2} A = \infty$

65. False; let $f(x) = (x^2 - 1)/(x - 1)$

67. False; let $f(x) = \tan x$

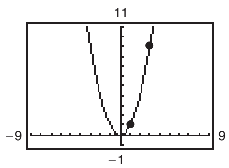
69. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{x^4}$, and let $c = 0$. $\lim_{x \to 0} \dfrac{1}{x^2} = \infty$ and $\lim_{x \to 0} \dfrac{1}{x^4} = \infty$, but $\lim_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{x^4}\right) = \lim_{x \to 0} \left(\dfrac{x^2 - 1}{x^4}\right) = -\infty \neq 0$.

71. Given $\lim_{x \to c} f(x) = \infty$, let $g(x) = 1$. Then $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$ by Theorem 1.15.

73. Answers will vary.

Review Exercises for Chapter 1 *(page 91)*

1. Calculus



Estimate: 8.3

3.

x	2.9	2.99	2.999	3
$f(x)$	−0.9091	−0.9901	−0.9990	?

x	3.001	3.01	3.1
$f(x)$	−1.0010	−1.0101	−1.1111

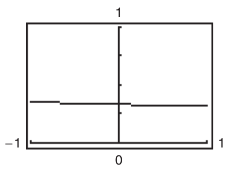
$\lim_{x \to 0} \dfrac{x - 3}{x^2 - 7x + 12} \approx -1.0000$

5. (a) 4 (b) 5 **7.** 5; Proof **9.** −3; Proof **11.** 36

13. $\sqrt{6} \approx 2.45$ **15.** 16 **17.** $\frac{4}{3}$ **19.** $-\frac{1}{4}$ **21.** $\frac{1}{2}$

23. −1 **25.** 0 **27.** $\sqrt{3}/2$ **29.** −3 **31.** −5

33.



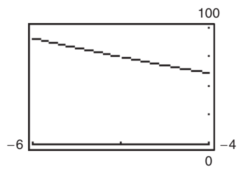
The graph has a hole at $x = 0$.

x	−0.1	−0.01	−0.001	0
$f(x)$	0.3352	0.3335	0.3334	?

x	0.001	0.01	0.1
$f(x)$	0.3333	0.3331	0.3315

$\lim_{x \to 0} \dfrac{\sqrt{2x + 9} - 3}{x} \approx 0.3333$; Actual limit is $\dfrac{1}{3}$.

35.



The graph has a hole at $x = -5$.

x	−5.1	−5.01	−5.001	−5
$f(x)$	76.51	75.15	75.02	?

x	−4.999	−4.99	−4.9
$f(x)$	74.99	74.85	73.51

$\lim_{x \to -5} \dfrac{x^3 + 125}{x + 5} \approx 75.00$; Actual limit is 75.

37. −39.2 m/sec **39.** $\frac{1}{6}$ **41.** $\frac{1}{4}$ **43.** 0

45. Limit does not exist. The limit as t approaches 1 from the left is 2, whereas the limit as t approaches 1 from the right is 1.

47. 3 **49.** Continuous for all real x

51. Nonremovable discontinuity at $x = 5$

53. Nonremovable discontinuities at $x = -1$ and $x = 1$
Removable discontinuity at $x = 0$

55. $c = -\frac{1}{2}$ **57.** Continuous for all real x

59. Continuous on $[4, \infty)$

61. Removable discontinuity at $x = 1$
Continuous on $(-\infty, 1) \cup (1, \infty)$

63. Proof **65.** (a) −4 (b) 4 (c) Limit does not exist.

67. $x = 0$ **69.** $x = \pm 3$ **71.** $x = \pm 8$ **73.** $-\infty$

75. $\frac{1}{3}$ **77.** $-\infty$ **79.** $\frac{4}{5}$ **81.** ∞

83. (a) \$14,117.65 (b) \$80,000.00 (c) \$720,000.00
(d) ∞

P.S. Problem Solving *(page 93)*

1. (a) Perimeter $\triangle PAO = 1 + \sqrt{(x^2 - 1)^2 + x^2} + \sqrt{x^4 + x^2}$
Perimeter $\triangle PBO = 1 + \sqrt{x^4 + (x - 1)^2} + \sqrt{x^4 + x^2}$

(b)

x	4	2	1
Perimeter $\triangle PAO$	33.0166	9.0777	3.4142
Perimeter $\triangle PBO$	33.7712	9.5952	3.4142
$r(x)$	0.9777	0.9461	1.0000

x	0.1	0.01
Perimeter $\triangle PAO$	2.0955	2.0100
Perimeter $\triangle PBO$	2.0006	2.0000
$r(x)$	1.0475	1.0050

1

3. (a) Area (hexagon) $= (3\sqrt{3})/2 \approx 2.5981$
Area (circle) $= \pi \approx 3.1416$
Area (circle) − Area (hexagon) ≈ 0.5435

(b) $A_n = (n/2) \sin(2\pi/n)$

(c)

n	6	12	24	48	96
A_n	2.5981	3.0000	3.1058	3.1326	3.1394

3.1416 or π

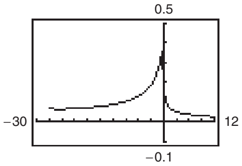
5. (a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$

(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

7. (a) Domain: $[-27, 1) \cup (1, \infty)$

(b)

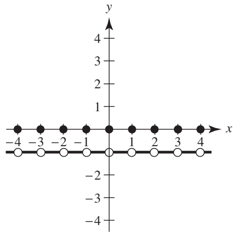


(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

The graph has a hole at $x = 1$.

9. (a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4

11.



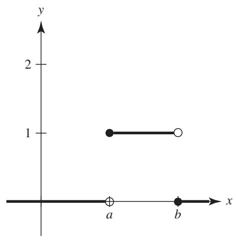
The graph jumps at every integer.

(a) $f(1) = 0, \quad f(0) = 0, \quad f\left(\frac{1}{2}\right) = -1, \quad f(-2.7) = -1$

(b) $\lim_{x\to 1^-} f(x) = -1, \quad \lim_{x\to 1^+} f(x) = -1, \quad \lim_{x\to 1/2} f(x) = -1$

(c) There is a discontinuity at each integer.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b

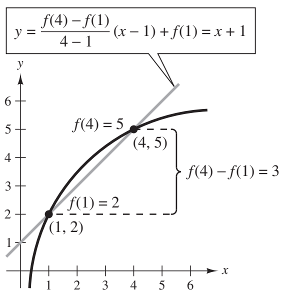
(d) The area under the graph of U and above the x-axis is 1.

Chapter 2

Section 2.1 *(page 103)*

1. $m_1 = 0, m_2 = 5/2$

3. (a)–(c)



5. $m = -5$

7. $m = 4$

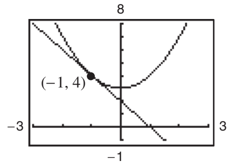
9. $m = 3$ **11.** $f'(x) = 0$ **13.** $f'(x) = -10$

15. $h'(s) = \frac{2}{3}$ **17.** $f'(x) = 2x + 1$ **19.** $f'(x) = 3x^2 - 12$

21. $f'(x) = \dfrac{-1}{(x - 1)^2}$ **23.** $f'(x) = \dfrac{1}{2\sqrt{x + 4}}$

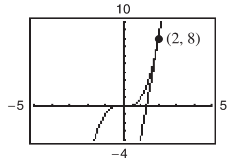
25. (a) Tangent line: $y = -2x + 2$

(b)



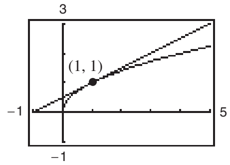
27. (a) Tangent line: $y = 12x - 16$

(b)



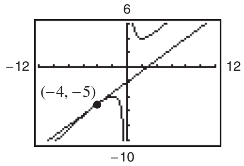
29. (a) Tangent line: $y = \frac{1}{2}x + \frac{1}{2}$

(b)



31. (a) Tangent line: $y = \frac{3}{4}x - 2$

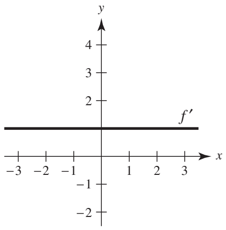
(b)



33. $y = 2x - 1$ **35.** $y = 3x - 2; y = 3x + 2$

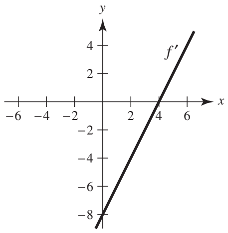
37. $y = -\frac{1}{2}x + \frac{3}{2}$

39.



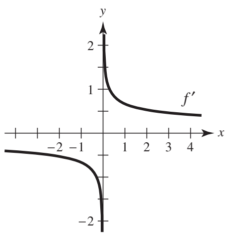
The slope of the graph of f is 1 for all x-values.

41.



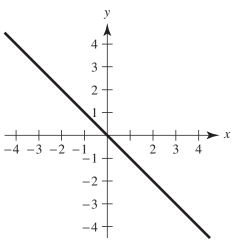
The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.

43.



The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.

45. Answers will vary.
Sample answer: $y = -x$

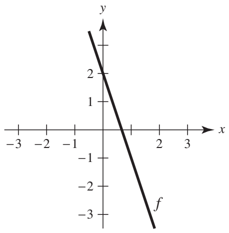


47. $g(4) = 5; g'(4) = -\frac{5}{3}$

49. $f(x) = 5 - 3x$
$c = 1$

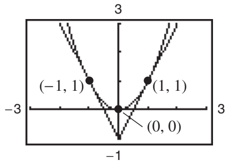
51. $f(x) = -x^2$
$c = 6$

53. $f(x) = -3x + 2$



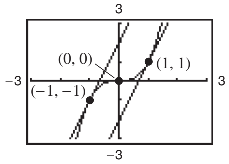
55. $y = 2x + 1$; $y = -2x + 9$

57. (a)



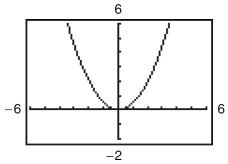
For this function, the slopes of the tangent lines are always distinct for different values of x.

(b)



For this function, the slopes of the tangent lines are sometimes the same.

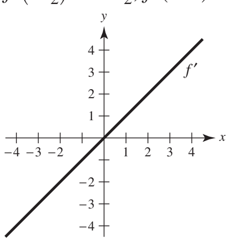
59. (a)



$f'(0) = 0, f'\left(\frac{1}{2}\right) = \frac{1}{2}, f'(1) = 1, f'(2) = 2$

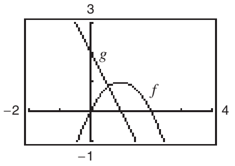
(b) $f'\left(-\frac{1}{2}\right) = -\frac{1}{2}, f'(-1) = -1, f'(-2) = -2$

(c)



(d) $f'(x) = x$

61.



$g(x) \approx f'(x)$

63. $f(2) = 4; f(2.1) = 3.99; f'(2) \approx -0.1$ **65.** 6 **67.** 4

69. $g(x)$ is not differentiable at $x = 0$.

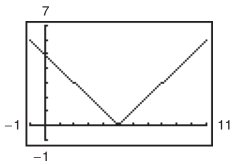
71. $f(x)$ is not differentiable at $x = 6$.

73. $h(x)$ is not differentiable at $x = -7$.

75. $(-\infty, 3) \cup (3, \infty)$ **77.** $(-\infty, -4) \cup (-4, \infty)$

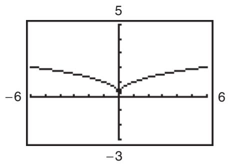
79. $(1, \infty)$

81.



$(-\infty, 5) \cup (5, \infty)$

83.



$(-\infty, 0) \cup (0, \infty)$

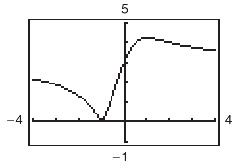
85. The derivative from the left is -1 and the derivative from the right is 1, so f is not differentiable at $x = 1$.

87. The derivatives from both the right and the left are 0, so $f'(1) = 0$.

89. f is differentiable at $x = 2$.

91. (a) $d = \left(3|m + 1|\right)/\sqrt{m^2 + 1}$

(b)



Not differentiable at $m = -1$

93. False. The slope is $\lim_{\Delta x \to 0} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

95. False. For example, $f(x) = |x|$. The derivative from the left and the derivative from the right both exist but are not equal.

97. Proof

Section 2.2 *(page 114)*

1. (a) $\frac{1}{2}$ (b) 3 **3.** 0 **5.** $7x^6$ **7.** $-5/x^6$

9. $1/(5x^{4/5})$ **11.** 1 **13.** $-4t + 3$ **15.** $2x + 12x^2$

17. $3t^2 + 10t - 3$ **19.** $\dfrac{\pi}{2}\cos\theta + \sin\theta$ **21.** $2x + \dfrac{1}{2}\sin x$

23. $-\dfrac{1}{x^2} - 3\cos x$

	Function	Rewrite	Derivative	Simplify
25.	$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\dfrac{5}{x^3}$
27.	$y = \dfrac{6}{(5x)^3}$	$y = \dfrac{6}{125}x^{-3}$	$y' = -\dfrac{18}{125}x^{-4}$	$y' = -\dfrac{18}{125x^4}$
29.	$y = \dfrac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\dfrac{1}{2}x^{-3/2}$	$y' = -\dfrac{1}{2x^{3/2}}$

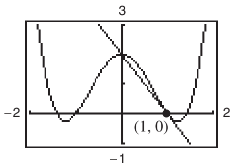
31. -2 **33.** 0 **35.** 8 **37.** 3 **39.** $2x + 6/x^3$

41. $2t + 12/t^4$ **43.** $8x + 3$ **45.** $(x^3 - 8)/x^3$

47. $3x^2 + 1$ **49.** $\dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$ **51.** $\dfrac{3}{\sqrt{x}} - 5\sin x$

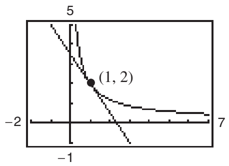
53. (a) $2x + y - 2 = 0$

(b)



55. (a) $3x + 2y - 7 = 0$

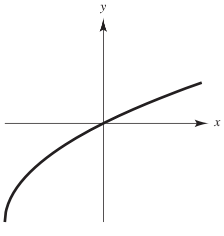
(b)



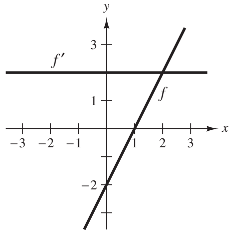
57. $(-1, 2), (0, 3), (1, 2)$ **59.** No horizontal tangents

61. (π, π) **63.** $k = -8$ **65.** $k = 3$ **67.** $k = 4/27$

69.

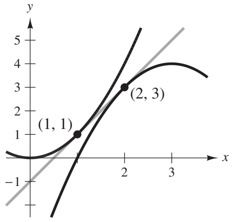


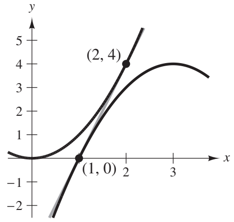
71. $g'(x) = f'(x)$ **73.** $g'(x) = -5f'(x)$

75. 

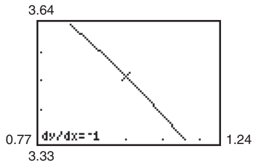
The rate of change of f is constant, and therefore f' is a constant function.

77. $y = 2x - 1$ $\quad$ $y = 4x - 4$

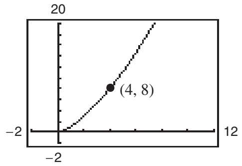




79. $f'(x) = 3 + \cos x \neq 0$ for all x. **81.** $x - 4y + 4 = 0$

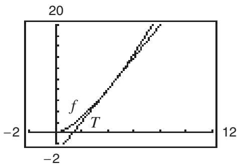
83. 

$f'(1)$ appears to be close to -1.
$f'(1) = -1$

85. (a) 

$(3.9, 7.7019)$,
$S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$
The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c) 

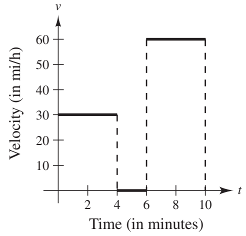
The approximation becomes less accurate.

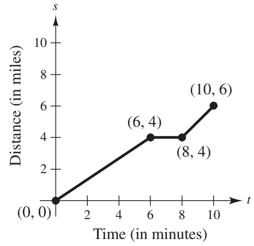
(d)

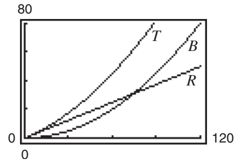
Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

87. False. Let $f(x) = x$ and $g(x) = x + 1$.
89. False. $dy/dx = 0$ **91.** True
93. Average rate: 4
Instantaneous rates:
$f'(1) = 4$; $f'(2) = 4$
95. Average rate: $\frac{1}{2}$
Instantaneous rates:
$f'(1) = 1$; $f'(2) = \frac{1}{4}$

97. (a) $s(t) = -16t^2 + 1362$; $v(t) = -32t$ (b) -48 ft/sec
(c) $s'(1) = -32$ ft/sec; $s'(2) = -64$ ft/sec
(d) $t = \dfrac{\sqrt{1362}}{4} \approx 9.226$ sec (e) -295.242 ft/sec
99. $v(5) = 71$ m/sec; $v(10) = 22$ m/sec
101. 

103. 

105. $V'(6) = 108\ \text{cm}^3/\text{cm}$
107. (a) $R(v) = 0.417v - 0.02$
(b) $B(v) = 0.0056v^2 + 0.001v + 0.04$
(c) $T(v) = 0.0056v^2 + 0.418v + 0.02$
(d) 

(e) $T'(v) = 0.0112v + 0.418$
$T'(40) = 0.866$
$T'(80) = 1.314$
$T'(100) = 1.538$

(f) Stopping distance increases at an increasing rate.
109. Proof **111.** $y = 2x^2 - 3x + 1$
113. $9x + y = 0$, $9x + 4y + 27 = 0$ **115.** $a = \frac{1}{3}$, $b = -\frac{4}{3}$
117. $f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.
$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.
119. Putnam Problem A2, 2010

Section 2.3 *(page 125)*

1. $2(2x^3 - 6x^2 + 3x - 6)$ **3.** $(1 - 5t^2)/(2\sqrt{t})$
5. $x^2(3\cos x - x\sin x)$ **7.** $(1 - x^2)/(x^2 + 1)^2$
9. $(1 - 5x^3)/[2\sqrt{x}(x^3 + 1)^2]$ **11.** $(x\cos x - 2\sin x)/x^3$
13. $f'(x) = (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4)$
$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$
$f'(0) = -20$
15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$
$f'(1) = -\dfrac{1}{4}$
17. $f'(x) = \cos x - x\sin x$
$f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	Rewrite	Differentiate	Simplify
19.	$y = \dfrac{x^2 + 3x}{7}$	$y = \dfrac{1}{7}x^2 + \dfrac{3}{7}x$	$y' = \dfrac{2}{7}x + \dfrac{3}{7}$	$y' = \dfrac{2x + 3}{7}$
21.	$y = \dfrac{6}{7x^2}$	$y = \dfrac{6}{7}x^{-2}$	$y' = -\dfrac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{3}{(x + 1)^2}$, $x \neq 1$ **27.** $(x^2 + 6x - 3)/(x + 3)^2$
29. $(3x + 1)/(2x^{3/2})$ **31.** $6s^2(s^3 - 2)$
33. $-(2x^2 - 2x + 3)/[x^2(x - 3)^2]$
35. $10x^4 - 8x^3 - 21x^2 - 10x - 30$
37. $-\dfrac{4xc^2}{(x^2 - c^2)^2}$ **39.** $t(t\cos t + 2\sin t)$

41. $-(t \sin t + \cos t)/t^2$ **43.** $-1 + \sec^2 x = \tan^2 x$

45. $\dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$ **47.** $\dfrac{3}{2} \sec x(\tan x - \sec x)$

49. $\cos x \cot^2 x$ **51.** $x(x \sec^2 x + 2 \tan x)$

53. $4x \cos x + (2 - x^2) \sin x$

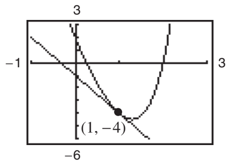
55. $\dfrac{2x^2 + 8x - 1}{(x + 2)^2}$ **57.** $\dfrac{1 - \sin \theta + \theta \cos \theta}{(1 - \sin \theta)^2}$

59. $y' = \dfrac{-2 \csc x \cot x}{(1 - \csc x)^2}, \quad -4\sqrt{3}$

61. $h'(t) = \sec t(t \tan t - 1)/t^2, \quad 1/\pi^2$

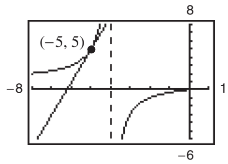
63. (a) $y = -3x - 1$

(b)



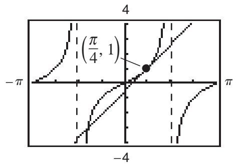
65. (a) $y = 4x + 25$

(b)



67. (a) $4x - 2y - \pi + 2 = 0$

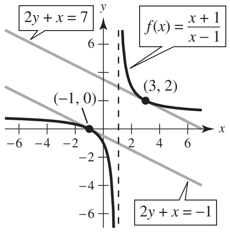
(b)



69. $2y + x - 4 = 0$

71. $25y - 12x + 16 = 0$ **73.** $(1, 1)$ **75.** $(0, 0), (2, 4)$

77. Tangent lines: $2y + x = 7$; $2y + x = -1$



79. $f(x) + 2 = g(x)$ **81.** (a) $p'(1) = 1$ (b) $q'(4) = -1/3$

83. $(18t + 5)/(2\sqrt{t})\ \text{cm}^2/\text{sec}$

85. (a) −$38.13 thousand/100 components

(b) −$10.37 thousand/100 components

(c) −$3.80 thousand/100 components

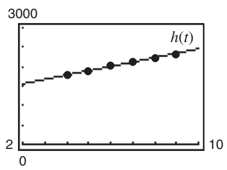
The cost decreases with increasing order size.

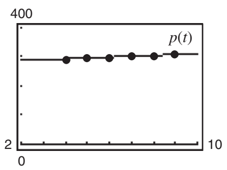
87. Proof

89. (a) $h(t) = 112.4t + 1332$

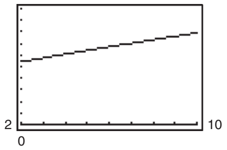
$p(t) = 2.9t + 282$

(b)





(c) $A = \dfrac{112.4t + 1332}{2.9t + 282}$



A represents the average health care expenditures per person (in thousands of dollars).

(d) $A'(t) = \dfrac{27{,}834}{8.41t^2 + 1635.6t + 79{,}524}$

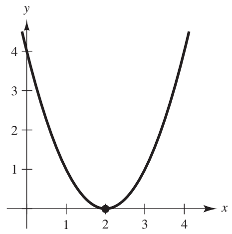
$A'(t)$ represents the rate of change of the average health care expenditures per person for the given year t.

91. $12x^2 + 12x - 6$ **93.** $3/\sqrt{x}$ **95.** $2/(x - 1)^3$

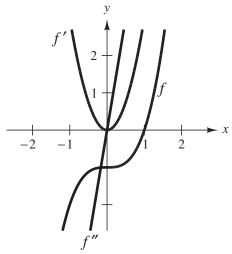
97. $2 \cos x - x \sin x$ **99.** $2x$ **101.** $1/\sqrt{x}$ **103.** 0

105. -10

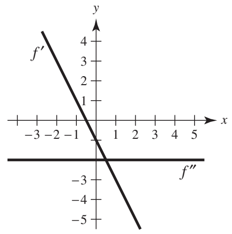
107. Answers will vary.
Sample answer:
$f(x) = (x - 2)^2$



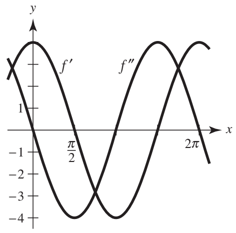
109.



111.



113.



115. $v(3) = 27$ m/sec

$a(3) = -6\ \text{m/sec}^2$

The speed of the object is decreasing.

117.

t	0	1	2	3	4
$s(t)$	0	57.75	99	123.75	132
$v(t)$	66	49.5	33	16.5	0
$a(t)$	−16.5	−16.5	−16.5	−16.5	−16.5

The average velocity on [0, 1] is 57.75, on [1, 2] is 41.25, on [2, 3] is 24.75, and on [3, 4] is 8.25.

119. $f^{(n)}(x) = n(n - 1)(n - 2) \cdots (2)(1) = n!$

121. (a) $f''(x) = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$

$f'''(x) = g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$

$f^{(4)}(x) = g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$

(b) $f^{(n)}(x) = g(x)h^{(n)}(x) + \dfrac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \dfrac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots + \dfrac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$

123. $n = 1$: $f'(x) = x \cos x + \sin x$

$n = 2$: $f'(x) = x^2 \cos x + 2x \sin x$

$n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$

$n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$

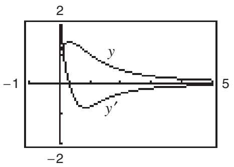
General rule: $f'(x) = x^n \cos x + nx^{(n-1)} \sin x$

125. $y' = -1/x^2,\ y'' = 2/x^3,$
$x^3y'' + 2x^2y' = x^3(2/x^3) + 2x^2(-1/x^2)$
$= 2 - 2 = 0$
127. $y' = 2\cos x,\ y'' = -2\sin x,$
$y'' + y = -2\sin x + 2\sin x + 3 = 3$
129. False. $dy/dx = f(x)g'(x) + g(x)f'(x)$ **131.** True
133. True **135.** $f'(x) = 2|x|;\ f''(0)$ does not exist.
137. Proof

Section 2.4 *(page 136)*

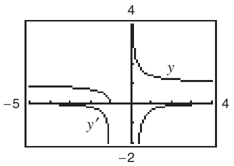
	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (5x - 8)^4$	$u = 5x - 8$	$y = u^4$
3.	$y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $12(4x - 1)^2$ **9.** $-108(4 - 9x)^3$ **11.** $-1/(2\sqrt{5 - t})$
13. $4x/\sqrt[3]{(6x^2 + 1)^2}$ **15.** $-x/\sqrt[4]{(9 - x^2)^3}$
17. $-1/(x - 2)^2$ **19.** $-2/(t - 3)^3$
21. $-3/[2\sqrt{(3x + 5)^3}]$ **23.** $2x(x - 2)^3(3x - 2)$
25. $\dfrac{1 - 2x^2}{\sqrt{1 - x^2}}$ **27.** $\dfrac{1}{\sqrt{(x^2 + 1)^3}}$
29. $\dfrac{-2(x + 5)(x^2 + 10x - 2)}{(x^2 + 2)^3}$ **31.** $\dfrac{-9(1 - 2v)^2}{(v + 1)^4}$
33. $20x(x^2 + 3)^9 + 2(x^2 + 3)^5 + 20x^2(x^2 + 3)^4 + 2x$
35. $(1 - 3x^2 - 4x^{3/2})/[2\sqrt{x}(x^2 + 1)^2]$



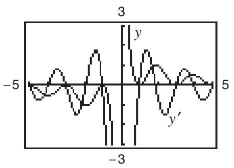
The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

37. $-\dfrac{\sqrt{\dfrac{x + 1}{x}}}{2x(x + 1)}$



y' has no zeros.

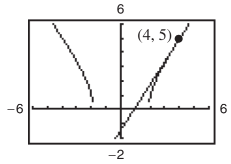
39. $-[\pi x \sin(\pi x) + \cos(\pi x) + 1]/x^2$



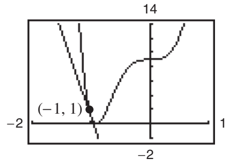
The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

41. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.
43. $-4\sin 4x$ **45.** $15\sec^2 3x$ **47.** $2\pi^2 x\cos(\pi x)^2$
49. $2\cos 4x$ **51.** $(-1 - \cos^2 x)/\sin^3 x$
53. $8\sec^2 x\tan x$ **55.** $10\tan 5\theta\sec^2 5\theta$
57. $\sin 2\theta\cos 2\theta = \dfrac{1}{2}\sin 4\theta$ **59.** $\dfrac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$
61. $\dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$ **63.** $2\sec^2 2x\cos(\tan 2x)$
65. $y' = \dfrac{x + 4}{\sqrt{x^2 + 8x}},\ \dfrac{5}{3}$ **67.** $f'(x) = \dfrac{-15x^2}{(x^3 - 2)^2},\ -\dfrac{3}{5}$

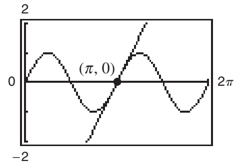
69. $f'(t) = \dfrac{-5}{(t - 1)^2},\ -5$ **71.** $y' = -12\sec^3 4x\tan 4x,\ 0$
73. (a) $8x - 5y - 7 = 0$
(b)



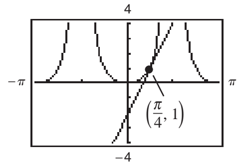
75. (a) $24x + y + 23 = 0$
(b)



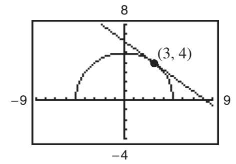
77. (a) $2x - y - 2\pi = 0$
(b)



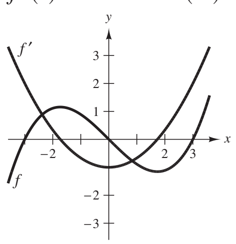
79. (a) $4x - y + (1 - \pi) = 0$
(b)



81. $3x + 4y - 25 = 0$

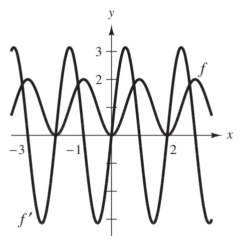


83. $\left(\dfrac{\pi}{6}, \dfrac{3\sqrt{3}}{2}\right), \left(\dfrac{5\pi}{6}, -\dfrac{3\sqrt{3}}{2}\right), \left(\dfrac{3\pi}{2}, 0\right)$ **85.** $2940(2 - 7x)^2$
87. $\dfrac{2}{(x - 6)^3}$ **89.** $2(\cos x^2 - 2x^2\sin x^2)$
91. $h''(x) = 18x + 6,\ 24$
93. $f''(x) = -4x^2\cos(x^2) - 2\sin(x^2),\ 0$
95.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

97.

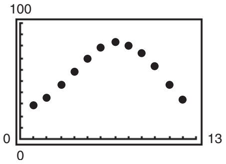


The zeros of f' correspond to the points where the graph of f has horizontal tangents.

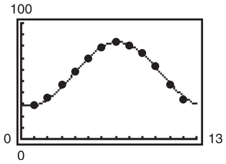
99. The rate of change of g is three times as fast as the rate of change of f.
101. (a) $g'(x) = f'(x)$ (b) $h'(x) = 2f'(x)$
(c) $r'(x) = -3f'(-3x)$ (d) $s'(x) = f'(x + 2)$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

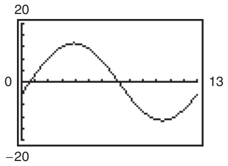
103. (a) $\frac{1}{2}$
(b) $s'(5)$ does not exist because g is not differentiable at 6.
105. (a) 1.461 (b) -1.016 **107.** 0.2 rad, 1.45 rad/sec

109. (a)


$T(t) = 56.1 + 27.6 \sin(0.48t - 1.86)$

(b)


The model is a good fit.

(c) $T'(t) \approx 13.25 \cos(0.48t - 1.86)$


(d) The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.)
The temperature changes most slowly around winter (Dec.–Feb.) and summer (Jun.–Aug.)
Yes. Explanations will vary.

111. (a) 0 bacteria per day (b) 177.8 bacteria per day
(c) 44.4 bacteria per day (d) 10.8 bacteria per day
(e) 3.3 bacteria per day
(f) The rate of change of the population is decreasing as time passes.

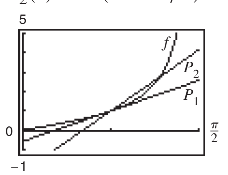
113. (a) $f'(x) = \beta \cos \beta x$
$f''(x) = -\beta^2 \sin \beta x$
$f'''(x) = -\beta^3 \cos \beta x$
$f^{(4)}(x) = \beta^4 \sin \beta x$
(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$
(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$
$f^{(2k-1)}(x) = (-1)^{k+1}\beta^{2k-1} \cos \beta x$

115. (a) $r'(1) = 0$ (b) $s'(4) = \frac{5}{8}$

117. (a) and (b) Proofs

119. $g'(x) = 3\left(\dfrac{3x - 5}{|3x - 5|}\right), \quad x \neq \dfrac{5}{3}$

121. $h'(x) = -|x|\sin x + \dfrac{x}{|x|}\cos x, \quad x \neq 0$

123. (a) $P_1(x) = 2(x - \pi/4) + 1$
$P_2(x) = 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$
(b)


(c) P_2

(d) The accuracy worsens as you move away from $x = \pi/4$.

125. False. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}(-1)$.

127. True **129.** Putnam Problem A1, 1967

Section 2.5 *(page 145)*

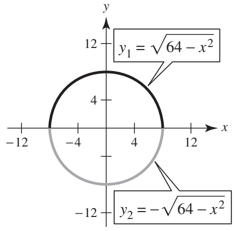
1. $-x/y$ **3.** $-\sqrt{y/x}$ **5.** $(y - 3x^2)/(2y - x)$

7. $(1 - 3x^2y^3)/(3x^3y^2 - 1)$

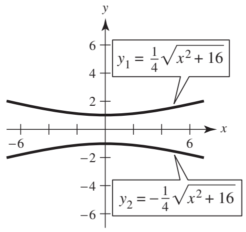
9. $(6xy - 3x^2 - 2y^2)/(4xy - 3x^2)$ **11.** $\cos/[4 \sin(2y)]$

13. $(\cos x - \tan y - 1)/(x \sec^2 y)$

15. $[y \cos(xy)]/[1 - x \cos(xy)]$

17. (a) $y_1 = \sqrt{64 - x^2}$; $y_2 = -\sqrt{64 - x^2}$
(b)


(c) $y' = \mp \dfrac{x}{\sqrt{64 - x^2}} = -\dfrac{x}{y}$ (d) $y' = -\dfrac{x}{y}$

19. (a) $y_1 = \dfrac{\sqrt{x^2 + 16}}{4}$; $y_2 = \dfrac{-\sqrt{x^2 + 16}}{4}$
(b)


(c) $y' = \dfrac{\pm x}{4\sqrt{x^2 + 16}} = \dfrac{x}{16y}$ (d) $y' = \dfrac{x}{16y}$

21. $-\dfrac{y}{x}, -\dfrac{1}{6}$ **23.** $\dfrac{98x}{y(x^2 + 49)^2}$, Undefined

25. $-\dfrac{y(y + 2x)}{x(x + 2y)}, \ -1$ **27.** $-\sin^2(x + y)$ or $-\dfrac{x^2}{x^2 + 1}, 0$

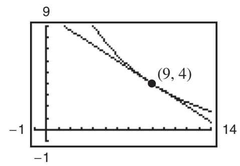
29. $-\dfrac{1}{2}$ **31.** 0 **33.** $y = -x + 7$ **35.** $y = -x + 2$

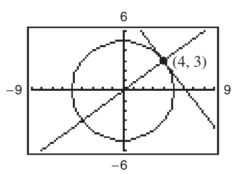
37. $y = \sqrt{3}x/6 + 8\sqrt{3}/3$ **39.** $y = -\frac{2}{11}x + \frac{30}{11}$

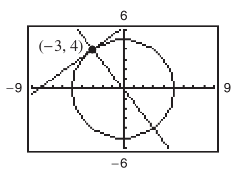
41. (a) $y = -2x + 4$ (b) Answers will vary.

43. $\cos^2 y, \ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}, \ \dfrac{1}{1 + x^2}$ **45.** $-4/y^3$

47. $-36/y^3$ **49.** $(3x)/(4y)$

51. $2x + 3y - 30 = 0$


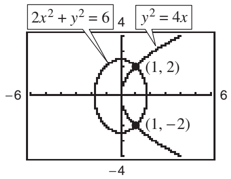
53. At (4, 3):
Tangent line: $4x + 3y - 25 = 0$
Normal line: $3x - 4y = 0$


At (−3, 4):
Tangent line: $3x - 4y + 25 = 0$
Normal line: $4x + 3y = 0$


55. $x^2 + y^2 = r^2 \Longrightarrow y' = -x/y \Longrightarrow y/x =$ slope of normal line. Then for (x_0, y_0) on the circle, $x_0 \neq 0$, an equation of the normal line is $y = (y_0/x_0)x$, which passes through the origin. If $x_0 = 0$, the normal line is vertical and passes through the origin.

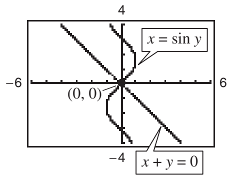
57. Horizontal tangents: $(-4, 0), (-4, 10)$
Vertical tangents: $(0, 5), (-8, 5)$

59.



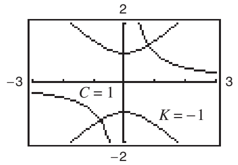
At $(1, 2)$:
Slope of ellipse: -1
Slope of parabola: 1
At $(1, -2)$:
Slope of ellipse: 1
Slope of parabola: -1

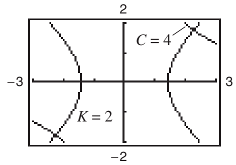
61.



At $(0, 0)$:
Slope of line: -1
Slope of sine curve: 1

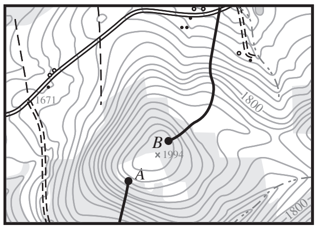
63. Derivatives: $\dfrac{dy}{dx} = -\dfrac{y}{x}, \dfrac{dy}{dx} = \dfrac{x}{y}$





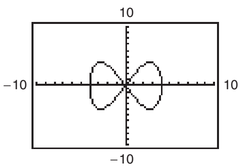
65. Answers will vary. In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form, it would be $y = (5 - x^2)/x$.

67.

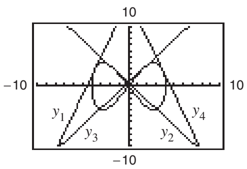


Use starting point B.

69. (a)



(b)

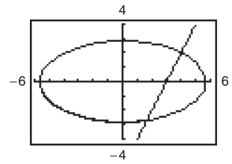


$y_1 = \frac{1}{3}\left[\left(\sqrt{7} + 7\right)x + \left(8\sqrt{7} + 23\right)\right]$
$y_2 = -\frac{1}{3}\left[\left(-\sqrt{7} + 7\right)x - \left(23 - 8\sqrt{7}\right)\right]$
$y_3 = -\frac{1}{3}\left[\left(\sqrt{7} - 7\right)x - \left(23 - 8\sqrt{7}\right)\right]$
$y_4 = -\frac{1}{3}\left[\left(\sqrt{7} + 7\right)x - \left(8\sqrt{7} + 23\right)\right]$

(c) $\left(\dfrac{8\sqrt{7}}{7}, 5\right)$

71. Proof **73.** $y = -\dfrac{\sqrt{3}}{2}x + 2\sqrt{3}, y = \dfrac{\sqrt{3}}{2}x - 2\sqrt{3}$

75. (a) $y = 2x - 6$
(b)



(c) $\left(\frac{28}{17}, -\frac{46}{17}\right)$

Section 2.6 *(page 153)*

1. (a) $\frac{3}{4}$ (b) 20 **3.** (a) $-\frac{5}{8}$ (b) $\frac{3}{2}$
5. (a) -8 cm/sec (b) 0 cm/sec (c) 8 cm/sec
7. (a) 12 ft/sec (b) 6 ft/sec (c) 3 ft/sec
9. In a linear function, if x changes at a constant rate, so does y. However, unless $a = 1$, y does not change at the same rate as x.
11. (a) 64π cm²/min (b) 256π cm²/min
13. (a) 972π in.³/min; $15{,}552\pi$ in.³/min
(b) If dr/dt is constant, dV/dt is proportional to r^2.
15. (a) 72 cm³/sec (b) 1800 cm³/sec
17. $8/(405\pi)$ ft/min **19.** (a) 12.5% (b) $\frac{1}{144}$ m/min
21. (a) $-\frac{7}{12}$ ft/sec; $-\frac{3}{2}$ ft/sec; $-\frac{48}{7}$ ft/sec
(b) $\frac{527}{24}$ ft²/sec (c) $\frac{1}{12}$ rad/sec
23. Rate of vertical change: $\frac{1}{5}$ m/sec
Rate of horizontal change: $-\sqrt{3}/15$ m/sec
25. (a) -750 mi/h (b) 30 min
27. $-50/\sqrt{85} \approx -5.42$ ft/sec
29. (a) $\frac{25}{3}$ ft/sec (b) $\frac{10}{3}$ ft/sec
31. (a) 12 sec (b) $\frac{1}{2}\sqrt{3}$ m (c) $\sqrt{5}\pi/120$ m/sec
33. Evaporation rate proportional to $S \Longrightarrow \dfrac{dV}{dt} = k(4\pi r^2)$

$V = \left(\dfrac{4}{3}\right)\pi r^3 \Longrightarrow \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$. So $k = \dfrac{dr}{dt}$.

35. 0.6 ohm/sec **37.** $\dfrac{dv}{dt} = \dfrac{16r}{v}\sec^2\theta\dfrac{d\theta}{dt}, \dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta\dfrac{dv}{dt}$

39. $\dfrac{2\sqrt{21}}{525} \approx 0.017$ rad/sec

41. (a) $\dfrac{200\pi}{3}$ ft/sec (b) 200π ft/sec
(c) About 427.43π ft/sec
43. About 84.9797 mi/h
45. (a) $\dfrac{dy}{dt} = 3\dfrac{dx}{dt}$ means that y changes three times as fast as x changes.
(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.
47. -18.432 ft/sec² **49.** About -97.96 m/sec

Review Exercises for Chapter 2 *(page 157)*

1. $f'(x) = 0$ **3.** $f'(x) = 2x - 4$ **5.** 5
7. f is differentiable at all $x \neq 3$. **9.** 0 **11.** $3x^2 - 22x$
13. $\dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$ **15.** $-\dfrac{4}{3t^3}$ **17.** $4 - 5\cos\theta$
19. $-3\sin\theta - (\cos\theta)/4$ **21.** -1 **23.** 0
25. (a) 50 vibrations/sec/lb (b) 33.33 vibrations/sec/lb

27. (a) $s(t) = -16t^2 - 30t + 600$
$v(t) = -32t - 30$
(b) -94 ft/sec
(c) $v'(1) = -62$ ft/sec; $v'(3) = -126$ ft/sec
(d) About 5.258 sec (e) About -198.256 ft/sec

29. $4(5x^3 - 15x^2 - 11x - 8)$ **31.** $\sqrt{x}\cos x + \sin x/(2\sqrt{x})$

33. $\dfrac{-(x^2+1)}{(x^2-1)^2}$ **35.** $\dfrac{4x^3\cos x + x^4\sin x}{\cos^2 x}$

37. $3x^2\sec x\tan x + 6x\sec x$ **39.** $-x\sin x$

41. $y = 4x + 10$ **43.** $y = -8x + 1$ **45.** $-48t$

47. $\frac{225}{4}\sqrt{x}$ **49.** $6\sec^2\theta\tan\theta$

51. $v(3) = 11$ m/sec; $a(3) = -6$ m/sec^2 **53.** $28(7x+3)^3$

55. $-\dfrac{2x}{(x^2+4)^2}$ **57.** $-45\sin(9x+1)$

59. $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$ **61.** $(36x+1)(6x+1)^4$

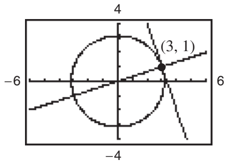
63. $\dfrac{3}{(x^2+1)^{3/2}}$ **65.** $\dfrac{-3x^2}{2\sqrt{1-x^3}}$; -2 **67.** $-\dfrac{8x}{(x^2+1)^2}$; 2

69. $-\csc 2x\cot 2x$; 0 **71.** $384(8x+5)$ **73.** $2\csc^2 x\cot x$

75. (a) $-18.667°$/h (b) $-7.284°$/h
(c) $-3.240°$/h (d) $-0.747°$/h

77. $-\dfrac{x}{y}$ **79.** $\dfrac{y(y^2-3x^2)}{x(x^2-3y^2)}$ **81.** $\dfrac{y\sin x + \sin y}{\cos x - x\cos y}$

83. Tangent line: $3x + y - 10 = 0$
Normal line: $x - 3y = 0$



85. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

87. 450π km/h

P.S. Problem Solving *(page 159)*

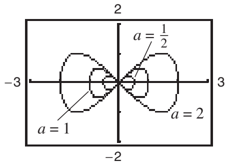
1. (a) $r = \frac{1}{2}$; $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$
(b) Center: $\left(0, \frac{5}{4}\right)$; $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$

3. $p(x) = 2x^3 + 4x^2 - 5$

5. (a) $y = 4x - 4$ (b) $y = -\frac{1}{4}x + \frac{9}{2}$; $\left(-\frac{9}{4}, \frac{81}{16}\right)$
(c) Tangent line: $y = 0$ (d) Proof
Normal line: $x = 0$

7. (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \end{cases}$ as separate equations.

(b) Answers will vary. Sample answer:



The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.

(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

9. (a) When the man is 90 ft from the light, the tip of his shadow is $112\frac{1}{2}$ ft from the light. The tip of the child's shadow is $111\frac{1}{9}$ ft from the light, so the man's shadow extends $1\frac{7}{18}$ ft beyond the child's shadow.
(b) When the man is 60 ft from the light, the tip of his shadow is 75 ft from the light. The tip of the child's shadow is $77\frac{7}{9}$ ft from the light, so the child's shadow extends $2\frac{7}{9}$ ft beyond the man's shadow.
(c) $d = 80$ ft
(d) Let x be the distance of the man from the light, and let s be the distance from the light to the tip of the shadow.
If $0 < x < 80$, then $ds/dt = -50/9$.
If $x > 80$, then $ds/dt = -25/4$.
There is a discontinuity at $x = 80$.

11. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec (b) 5 sec; 73.5 ft
$a(t) = -\frac{27}{5}$ ft/sec^2
(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

13. Proof. The graph of L is a line passing through the origin $(0, 0)$.

15. (a) j would be the rate of change of acceleration.
(b) $j = 0$. Acceleration is constant, so there is no change in acceleration.
(c) a: position function, d: velocity function,
b: acceleration function, c: jerk function

Chapter 3

Section 3.1 *(page 167)*

1. $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum);
2, absolute minimum (and relative minimum);
3, absolute maximum (and relative maximum)

11. $x = 0, x = 2$ **13.** $t = 8/3$ **15.** $x = \pi/3, \pi, 5\pi/3$

17. Minimum: $(2, 1)$
Maximum: $(-1, 4)$

19. Minimum: $(2, -8)$
Maximum: $(6, 24)$

21. Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

23. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

25. Minimum: $(0, 0)$
Maxima: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

27. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

29. Minimum: $(-1, -1)$
Maximum: $(3, 3)$

31. Minimum value is -2 for $-2 \le x < -1$.
Maximum: $(2, 2)$

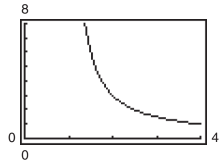
33. Minimum: $(3\pi/2, -1)$
Maximum: $(5\pi/6, 1/2)$

35. Minimum: $(\pi, -3)$
Maxima: $(0, 3)$ and $(2\pi, 3)$

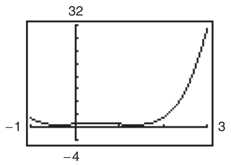
37. (a) Minimum: $(0, -3)$;
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

39. (a) Minimum: $(1, -1)$;
Maximum: $(-1, 3)$
(b) Maximum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

41.

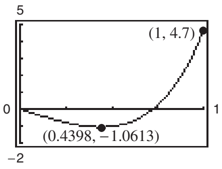


Minimum: $(4, 1)$

43. 

Minima: $\left(\dfrac{-\sqrt{3}+1}{2}, \dfrac{3}{4}\right)$ and $\left(\dfrac{\sqrt{3}+1}{2}, \dfrac{3}{4}\right)$

Maximum: $(3, 31)$

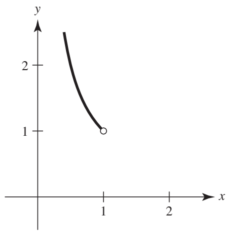
45. (a) 

(b) Minimum: $(0.4398, -1.0613)$

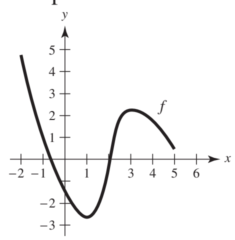
47. Maximum: $\left|f''\left(\sqrt[3]{-10+\sqrt{108}}\right)\right| = f''\left(\sqrt{3}-1\right) \approx 1.47$

49. Maximum: $\left|f^{(4)}(0)\right| = \frac{56}{81}$

51. Answers will vary. Sample answer: Let $f(x) = 1/x$. f is continuous on $(0, 1)$ but does not have a maximum or minimum.



53. Answers will vary. Sample answer:



55. (a) Yes (b) No **57.** (a) No (b) Yes

59. Maximum: $P(12) = 72$; No. P is decreasing for $I > 12$.

61. $\theta = \operatorname{arcsec}\sqrt{3} \approx 0.9553$ rad

63. True **65.** True **67.** Proof

69. Putnam Problem B3, 2004

Section 3.2 *(page 174)*

1. $f(-1) = f(1) = 1$; f is not continuous on $[-1, 1]$.

3. $f(0) = f(2) = 0$; f is not differentiable on $(0, 2)$.

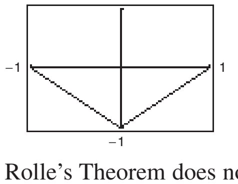
5. $(2, 0), (-1, 0)$; $f'\left(\frac{1}{2}\right) = 0$ **7.** $(0, 0), (-4, 0)$; $f'\left(-\frac{8}{3}\right) = 0$

9. $f'\left(\frac{3}{2}\right) = 0$ **11.** $f'\left(\dfrac{6-\sqrt{3}}{3}\right) = 0$; $f'\left(\dfrac{6+\sqrt{3}}{3}\right) = 0$

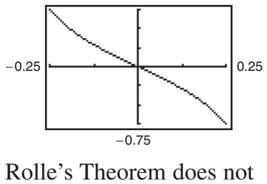
13. Not differentiable at $x = 0$ **15.** $f'\left(-2+\sqrt{5}\right) = 0$

17. $f'\left(\dfrac{\pi}{2}\right) = 0$; $f'\left(\dfrac{3\pi}{2}\right) = 0$ **19.** $f'\left(\dfrac{\pi}{6}\right) = 0$

21. Not continuous on $[0, \pi]$

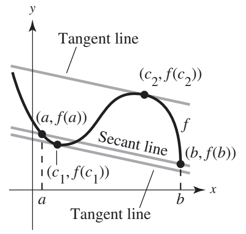
23. 

Rolle's Theorem does not apply.

25. 

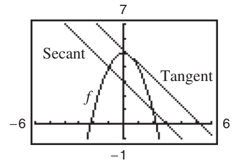
Rolle's Theorem does not apply.

27. (a) $f(1) = f(2) = 38$
(b) Velocity $= 0$ for some t in $(1, 2)$; $t = \frac{3}{2}$ sec

29. 

31. The function is not continuous on $[0, 6]$.

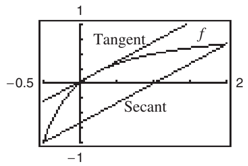
33. The function is not continuous on $[0, 6]$.

35. (a) Secant line: $x + y - 3 = 0$ (b) $c = \frac{1}{2}$
(c) Tangent line: $4x + 4y - 21 = 0$
(d) 

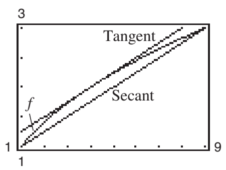
37. $f'(-1/2) = -1$ **39.** $f'\left(1/\sqrt{3}\right) = 3$, $f'\left(-1/\sqrt{3}\right) = 3$

41. $f'\left(\frac{8}{27}\right) = 1$ **43.** f is not differentiable at $x = -\frac{1}{2}$.

45. $f'(\pi/2) = 0$

47. (a)–(c) 

(b) $y = \frac{2}{3}(x - 1)$
(c) $y = \frac{1}{3}\left(2x + 5 - 2\sqrt{6}\right)$

49. (a)–(c) 

(b) $y = \frac{1}{4}x + \frac{3}{4}$
(c) $y = \frac{1}{4}x + 1$

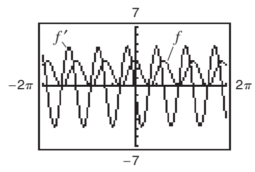
51. (a) -14.7 m/sec (b) 1.5 sec

53. No. Let $f(x) = x^2$ on $[-1, 2]$.

55. No. $f(x)$ is not continuous on $[0, 1]$. So it does not satisfy the hypothesis of Rolle's Theorem.

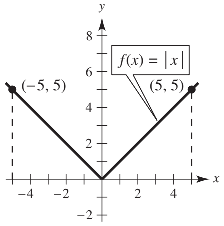
57. By the Mean Value Theorem, there is a time when the speed of the plane must equal the average speed of 454.5 miles/hour. The speed was 400 miles/hour when the plane was accelerating to 454.5 miles/hour and decelerating from 454.5 miles/hour.

59. Proof

61. (a) 

(b) Yes; yes
(c) Because $f(-1) = f(1) = 0$, Rolle's Theorem applies on $[-1, 1]$. Because $f(1) = 0$ and $f(2) = 3$, Rolle's Theorem does not apply on $[1, 2]$.
(d) $\displaystyle\lim_{x\to 3^-} f'(x) = 0$; $\displaystyle\lim_{x\to 3^+} f'(x) = 0$

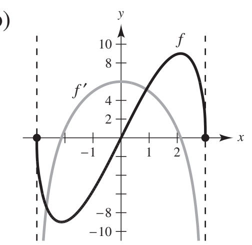
63.



65–67. Proofs **69.** $f(x) = 5$ **71.** $f(x) = x^2 - 1$
73. False. f is not continuous on $[-1, 1]$. **75.** True
77–85. Proofs

Section 3.3 *(page 183)*

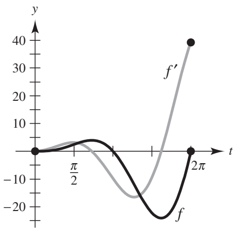
1. (a) $(0, 6)$ (b) $(6, 8)$
3. Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$
5. Increasing on $(-\infty, -2)$ and $(2, \infty)$; Decreasing on $(-2, 2)$
7. Increasing on $(-\infty, -1)$; Decreasing on $(-1, \infty)$
9. Increasing on $(1, \infty)$; Decreasing on $(-\infty, 1)$
11. Increasing on $(-2\sqrt{2}, 2\sqrt{2})$;
Decreasing on $(-4, -2\sqrt{2})$ and $(2\sqrt{2}, 4)$
13. Increasing on $(0, \pi/2)$ and $(3\pi/2, 2\pi)$;
Decreasing on $(\pi/2, 3\pi/2)$
15. Increasing on $(0, 7\pi/6)$ and $(11\pi/6, 2\pi)$;
Decreasing on $(7\pi/6, 11\pi/6)$
17. (a) Critical number: $x = 2$
(b) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$
(c) Relative minimum: $(2, -4)$
19. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 5)$
21. (a) Critical numbers: $x = -2, 1$
(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$;
Decreasing on $(-2, 1)$
(c) Relative maximum: $(-2, 20)$;
Relative minimum: $(1, -7)$
23. (a) Critical numbers: $x = -\frac{5}{3}, 1$
(b) Increasing on $\left(-\infty, -\frac{5}{3}\right)$, $(1, \infty)$;
Decreasing on $\left(-\frac{5}{3}, 1\right)$
(c) Relative maximum: $\left(-\frac{5}{3}, \frac{256}{27}\right)$;
Relative minimum: $(1, 0)$
25. (a) Critical numbers: $x = \pm 1$
(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$;
Decreasing on $(-1, 1)$
(c) Relative maximum: $\left(-1, \frac{4}{5}\right)$; Relative minimum: $\left(1, -\frac{4}{5}\right)$
27. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, \infty)$
(c) No relative extrema
29. (a) Critical number: $x = -2$
(b) Increasing on $(-2, \infty)$; Decreasing on $(-\infty, -2)$
(c) Relative minimum: $(-2, 0)$
31. (a) Critical number: $x = 5$
(b) Increasing on $(-\infty, 5)$; Decreasing on $(5, \infty)$
(c) Relative maximum: $(5, 5)$
33. (a) Critical numbers: $x = \pm\sqrt{2}/2$; Discontinuity: $x = 0$
(b) Increasing on $(-\infty, -\sqrt{2}/2)$ and $(\sqrt{2}/2, \infty)$;
Decreasing on $(-\sqrt{2}/2, 0)$ and $(0, \sqrt{2}/2)$
(c) Relative maximum: $(-\sqrt{2}/2, -2\sqrt{2})$;
Relative minimum: $(\sqrt{2}/2, 2\sqrt{2})$
35. (a) Critical number: $x = 0$; Discontinuities: $x = \pm 3$
(b) Increasing on $(-\infty, -3)$ and $(-3, 0)$;
Decreasing on $(0, 3)$ and $(3, \infty)$
(c) Relative maximum: $(0, 0)$
37. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$
(c) Relative maximum: $(0, 4)$
39. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 4)$
41. (a) Critical numbers: $x = \pi/6, 5\pi/6$;
Increasing on $(0, \pi/6)$, $(5\pi/6, 2\pi)$;
Decreasing on $(\pi/6, 5\pi/6)$
(b) Relative maximum: $(\pi/6, (\pi + 6\sqrt{3})/12)$;
Relative minimum: $(5\pi/6, (5\pi - 6\sqrt{3})/12)$
43. (a) Critical numbers: $x = \pi/4, 5\pi/4$;
Increasing on $(0, \pi/4)$, $(5\pi/4, 2\pi)$;
Decreasing on $(\pi/4, 5\pi/4)$
(b) Relative maximum: $(\pi/4, \sqrt{2})$;
Relative minimum: $(5\pi/4, -\sqrt{2})$
45. (a) Critical numbers:
$x = \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$;
Increasing on $(\pi/4, \pi/2)$, $(3\pi/4, \pi)$, $(5\pi/4, 3\pi/2)$, $(7\pi/4, 2\pi)$;
Decreasing on $(0, \pi/4)$, $(\pi/2, 3\pi/4)$, $(\pi, 5\pi/4)$, $(3\pi/2, 7\pi/4)$;
(b) Relative maxima: $(\pi/2, 1)$, $(\pi, 1)$, $(3\pi/2, 1)$;
Relative minima: $(\pi/4, 0)$, $(3\pi/4, 0)$, $(5\pi/4, 0)$, $(7\pi/4, 0)$
47. (a) Critical numbers: $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$;
Increasing on $(0, \pi/2)$, $(7\pi/6, 3\pi/2)$, $(11\pi/6, 2\pi)$;
Decreasing on $(\pi/2, 7\pi/6)$, $(3\pi/2, 11\pi/6)$
(b) Relative maxima: $(\pi/2, 2)$, $(3\pi/2, 0)$;
Relative minima: $(7\pi/6, -1/4)$, $(11\pi/6, -1/4)$
49. (a) $f'(x) = 2(9 - 2x^2)/\sqrt{9 - x^2}$
(b)



(c) Critical numbers: $x = \pm 3\sqrt{2}/2$
(d) $f' > 0$ on $(-3\sqrt{2}/2, 3\sqrt{2}/2)$;
$f' < 0$ on $(-3, -3\sqrt{2}/2)$, $(3\sqrt{2}/2, 3)$
f is increasing when f' is positive and decreasing when f' is negative.

51. (a) $f'(t) = t(t\cos t + 2\sin t)$

(b)

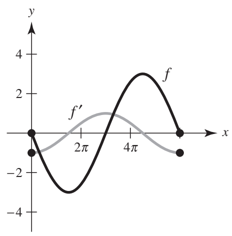


(c) Critical numbers: $t = 2.2889, 5.0870$

(d) $f' > 0$ on $(0, 2.2889), (5.0870, 2\pi)$;
$f' < 0$ on $(2.2889, 5.0870)$
f is increasing when f' is positive and decreasing when f' is negative.

53. (a) $f'(x) = -\cos(x/3)$

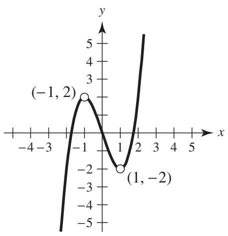
(b)



(c) Critical numbers: $x = 3\pi/2, 9\pi/2$

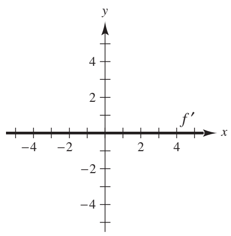
(d) $f' > 0$ on $\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$;
$f' < 0$ on $\left(0, \frac{3\pi}{2}\right), \left(\frac{9\pi}{2}, 6\pi\right)$
f is increasing when f' is positive and decreasing when f' is negative.

55. $f(x)$ is symmetric with respect to the origin.
Zeros: $(0, 0), (\pm\sqrt{3}, 0)$

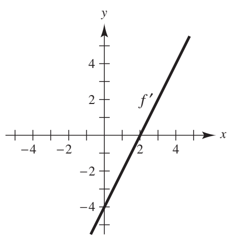


$g(x)$ is continuous on $(-\infty, \infty)$, and $f(x)$ has holes at $x = 1$ and $x = -1$.

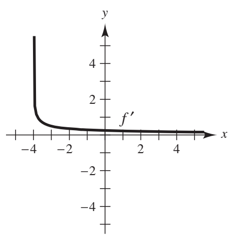
57.



59.

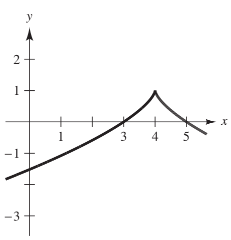


61.



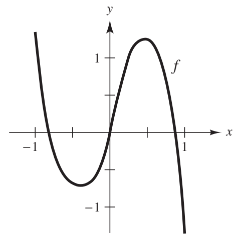
63. $g'(0) < 0$ **65.** $g'(-6) < 0$ **67.** $g'(0) > 0$

69. Answers will vary. Sample answer:



71. $(5, f(5))$ is a relative minimum.

73. (a)



(b) Critical numbers: $x \approx -0.40$ and $x \approx 0.48$
(c) Relative maximum: $(0.48, 1.25)$;
Relative minimum: $(-0.40, 0.75)$

75. (a) $s'(t) = 9.8(\sin\theta)t$; speed $= |9.8(\sin\theta)t|$
(b)

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$s'(t)$	0	$4.9\sqrt{2}t$	$4.9\sqrt{3}t$	$9.8t$	$4.9\sqrt{3}t$	$4.9\sqrt{2}t$	0

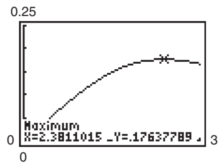
The speed is maximum at $\theta = \pi/2$.

77. (a)

t	0	0.5	1	1.5	2	2.5	3
$C(t)$	0	0.055	0.107	0.148	0.171	0.176	0.167

$t = 2.5$ h

(b)



$t \approx 2.38$ h (c) $t \approx 2.38$ h

79. $r = 2R/3$

81. (a) $v(t) = 6 - 2t$ (b) $[0, 3)$ (c) $(3, \infty)$ (d) $t = 3$

83. (a) $v(t) = 3t^2 - 10t + 4$
(b) $\left[0, (5 - \sqrt{13})/3\right)$ and $\left((5 + \sqrt{13})/3, \infty\right)$
(c) $\left(\frac{5 - \sqrt{13}}{3}, \frac{5 + \sqrt{13}}{3}\right)$ (d) $t = \frac{5 \pm \sqrt{13}}{3}$

85. Answers will vary.

87. (a) Minimum degree: 3
(b)
$$a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$$
$$a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 2$$
$$3a_3(0)^2 + 2a_2(0) + a_1 = 0$$
$$3a_3(2)^2 + 2a_2(2) + a_1 = 0$$
(c) $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$

89. (a) Minimum degree: 4
(b)
$$a_4(0)^4 + a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$$
$$a_4(2)^4 + a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 4$$
$$a_4(4)^4 + a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 = 0$$
$$4a_4(0)^3 + 3a_3(0)^2 + 2a_2(0) + a_1 = 0$$
$$4a_4(2)^3 + 3a_3(2)^2 + 2a_2(2) + a_1 = 0$$
$$4a_4(4)^3 + 3a_3(4)^2 + 2a_2(4) + a_1 = 0$$
(c) $f(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2$

91. True **93.** False. Let $f(x) = x^3$.

95. False. Let $f(x) = x^3$. There is a critical number at $x = 0$, but not a relative extremum.

97–99. Proofs **101.** Putnam Problem A3, 2003

Section 3.4 *(page 192)*

1. $f' > 0, f'' < 0$ **3.** Concave upward: $(-\infty, \infty)$

5. Concave upward: $(-\infty, 2)$; Concave downward: $(2, \infty)$

7. Concave upward: $(-\infty, -2), (2, \infty)$;
Concave downward: $(-2, 2)$

9. Concave upward: $(-\infty, -1), (1, \infty)$;
Concave downward: $(-1, 1)$

11. Concave upward: $(-2, 2)$;
Concave downward: $(-\infty, -2), (2, \infty)$

13. Concave upward: $(-\pi/2, 0)$; Concave downward: $(0, \pi/2)$

15. Point of inflection: $(2, 8)$; Concave downward: $(-\infty, 2)$;
Concave upward: $(2, \infty)$

17. Points of inflection: $(-2, -8)$, $(0, 0)$;
Concave upward: $(-\infty, -2)$, $(0, \infty)$;
Concave downward: $(-2, 0)$

19. Points of inflection: $(2, -16), (4, 0)$;
Concave upward: $(-\infty, 2), (4, \infty)$;
Concave downward: $(2, 4)$

21. Concave upward: $(-3, \infty)$

23. Points of inflection: $\left(-\sqrt{3}/3, 3\right), \left(\sqrt{3}/3, 3\right)$;
Concave upward: $\left(-\infty, -\sqrt{3}/3\right), \left(\sqrt{3}/3, \infty\right)$;
Concave downward: $\left(-\sqrt{3}/3, \sqrt{3}/3\right)$

25. Point of inflection: $(2\pi, 0)$;
Concave upward: $(2\pi, 4\pi)$; Concave downward: $(0, 2\pi)$

27. Concave upward: $(0, \pi)$, $(2\pi, 3\pi)$;
Concave downward: $(\pi, 2\pi)$, $(3\pi, 4\pi)$

29. Points of inflection: $(\pi, 0), (1.823, 1.452), (4.46, -1.452)$
Concave upward: $(1.823, \pi), (4.46, 2\pi)$
Concave downward: $(0, 1.823), (\pi, 4.46)$

31. Relative maximum: $(3, 9)$

33. Relative maximum: $(0, 3)$; Relative minimum: $(2, -1)$

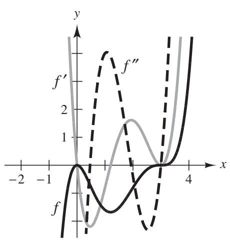
35. Relative minimum: $(3, -25)$

37. Relative minimum: $(0, -3)$

39. Relative maximum: $(-2, -4)$; Relative minimum: $(2, 4)$

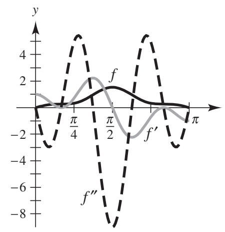
41. No relative extrema, because f is nonincreasing.

43. (a) $f'(x) = 0.2x(x - 3)^2(5x - 6)$;
$f''(x) = 0.4(x - 3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$;
Relative minimum: $(1.2, -1.6796)$;
Points of inflection: $(0.4652, -0.7048)$,
$(1.9348, -0.9048)$, $(3, 0)$
(c)



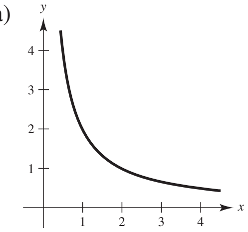
f is increasing when f' is positive and decreasing when f' is negative. f is concave upward when f'' is positive and concave downward when f'' is negative.

45. (a) $f'(x) = \cos x - \cos 3x + \cos 5x$;
$f''(x) = -\sin x + 3 \sin 3x - 5 \sin 5x$
(b) Relative maximum: $(\pi/2, 1.53333)$;
Points of inflection: $(\pi/6, 0.2667), (1.1731, 0.9637)$,
$(1.9685, 0.9637), (5\pi/6, 0.2667)$
(c)

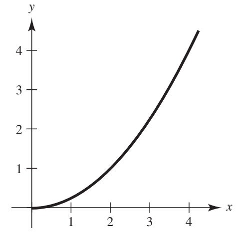


f is increasing when f' is positive and decreasing when f' is negative. f is concave upward when f'' is positive and concave downward when f'' is negative.

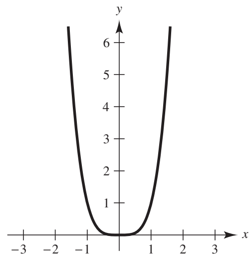
47. (a)



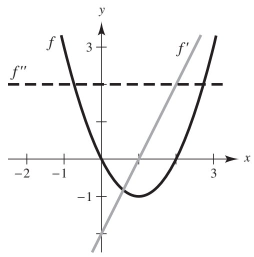
(b)



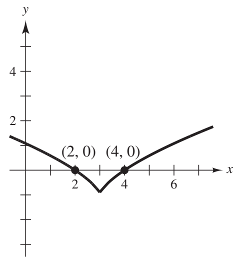
49. Answers will vary. Sample answer: $f(x) = x^4$; $f''(0) = 0$, but $(0, 0)$ is not a point of inflection.



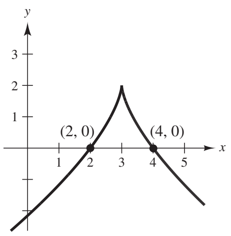
51.



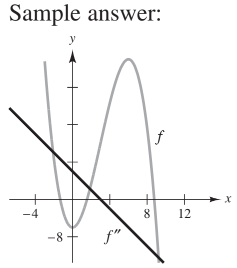
53.



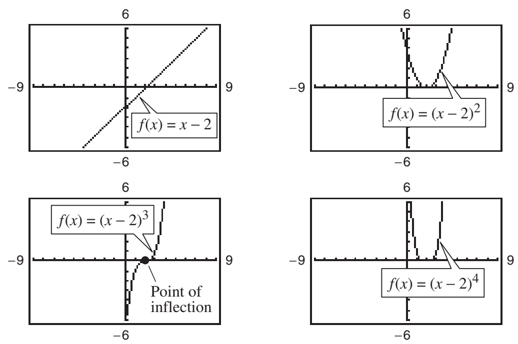
55.



57. Sample answer:



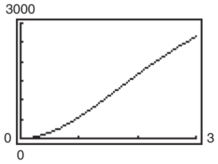
59. (a) $f(x) = (x - 2)^n$ has a point of inflection at $(2, 0)$ if n is odd and $n \geq 3$.



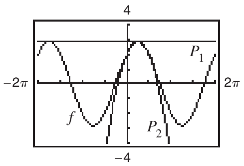
(b) Proof

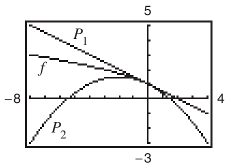
61. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$
63. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown
65. $x = 100$ units
67. (a)

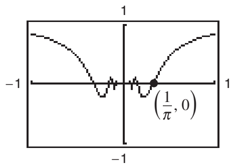
t	0.5	1	1.5	2	2.5	3
S	151.5	555.6	1097.6	1666.7	2193.0	2647.1

$1.5 < t < 2$
(b)


$t \approx 1.5$
(c) About 1.633 yr

69. $P_1(x) = 2\sqrt{2}$
$P_2(x) = 2\sqrt{2} - \sqrt{2}(x - \pi/4)^2$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.


71. $P_1(x) = 1 - x/2$
$P_2(x) = 1 - x/2 - x^2/8$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.


73.


75. True

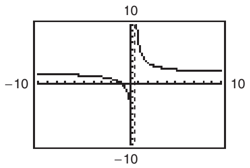
77. False. f is concave upward at $x = c$ if $f''(c) > 0$.
79. Proof

Section 3.5 *(page 202)*

1. f **2.** c **3.** d **4.** a **5.** b **6.** e
7.

x	10^0	10^1	10^2	10^3
$f(x)$	7	2.2632	2.0251	2.0025

x	10^4	10^5	10^6
$f(x)$	2.0003	2.0000	2.0000

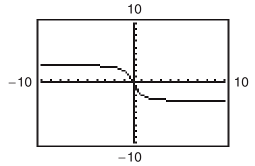


$$\lim_{x\to\infty} \frac{4x + 3}{2x - 1} = 2$$

9.

x	10^0	10^1	10^2	10^3
$f(x)$	-2	-2.9814	-2.9998	-3.0000

x	10^4	10^5	10^6
$f(x)$	-3.0000	-3.0000	-3.0000

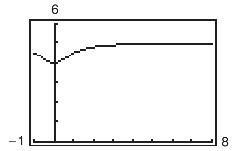


$$\lim_{x\to\infty} \frac{-6x}{\sqrt{4x^2 + 5}} = -3$$

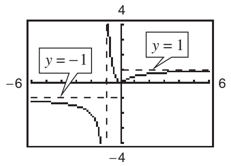
11.

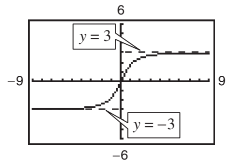
x	10^0	10^1	10^2	10^3
$f(x)$	4.5000	4.9901	4.9999	5.0000

x	10^4	10^5	10^6
$f(x)$	5.0000	5.0000	5.0000



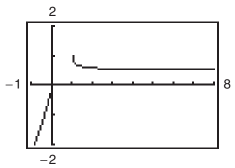
$$\lim_{x\to\infty} \left(5 - \frac{1}{x^2 + 1}\right) = 5$$

13. (a) ∞ (b) 5 (c) 0 **15.** (a) 0 (b) 1 (c) ∞
17. (a) 0 (b) $-\frac{2}{3}$ (c) $-\infty$ **19.** 4 **21.** $\frac{2}{3}$ **23.** 0
25. $-\infty$ **27.** -1 **29.** -2 **31.** $\frac{1}{2}$ **33.** ∞
35. 0 **37.** 0
39.


41.


43. 1 **45.** 0 **47.** $\frac{1}{6}$
49.

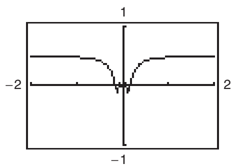
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.000	0.513	0.501	0.500	0.500	0.500	0.500



$$\lim_{x\to\infty} \left[x - \sqrt{x(x-1)}\right] = \frac{1}{2}$$

51.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500

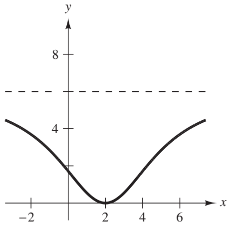


The graph has a hole at $x = 0$.
$$\lim_{x\to\infty} x \sin\frac{1}{2x} = \frac{1}{2}$$

53. As x becomes large, $f(x)$ approaches 4.

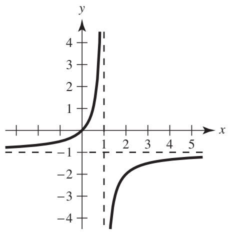
55. Answers will vary. Sample answer: Let

$$f(x) = \frac{-6}{0.1(x-2)^2+1} + 6.$$

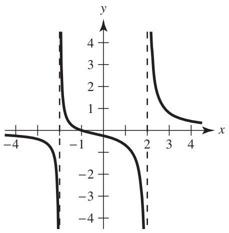


57. (a) 5 (b) -5

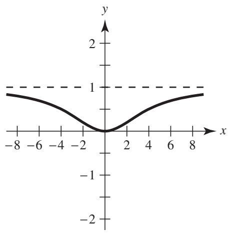
59.



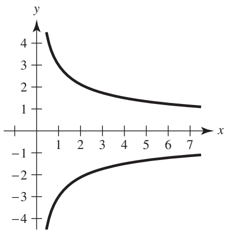
61.



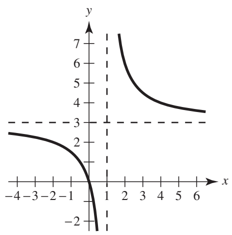
63.



65.



67.



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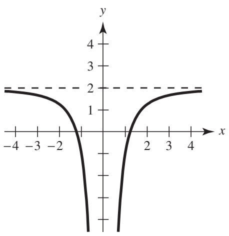

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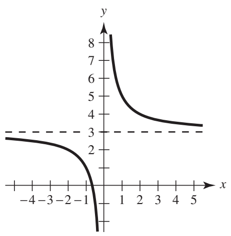

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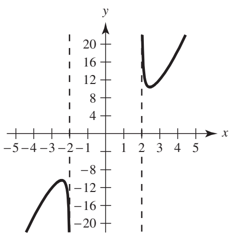
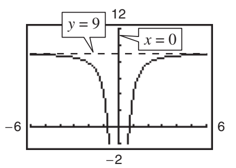
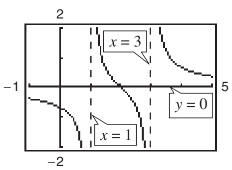


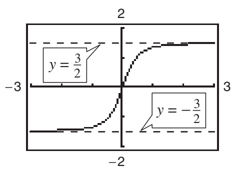
75.



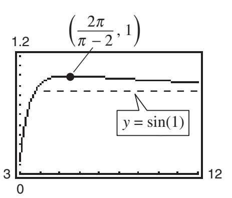
77.



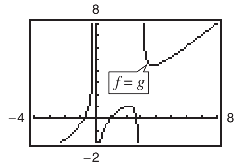
79.



81.

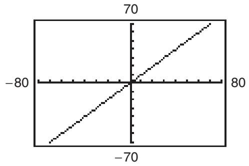


83. (a)



(b) Proof

(c)

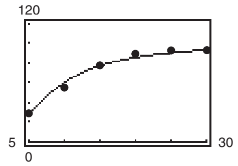


Slant asymptote: $y = x$

85. 100%

87. $\lim_{t\to\infty} N(t) = +\infty$; $\lim_{t\to\infty} E(t) = c$

89. (a)



(b) Yes. $\lim_{t\to\infty} S = \frac{100}{1} = 100$

91. (a) $\lim_{x\to\infty} f(x) = 2$

(b) $x_1 = \sqrt{\frac{4-2\varepsilon}{\varepsilon}}$, $x_2 = -\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

(c) $\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$ (d) $-\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

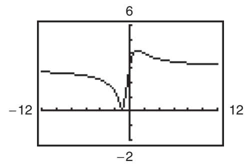
93. (a) Answers will vary. $M = \frac{5\sqrt{33}}{11}$

(b) Answers will vary. $M = \frac{29\sqrt{177}}{59}$

95–97. Proofs

99. (a) $d(m) = \frac{|3m+3|}{\sqrt{m^2+1}}$

(b)



(c) $\lim_{m\to\infty} d(m) = 3$;
$\lim_{m\to-\infty} d(m) = 3$;
As m approaches $\pm\infty$, the distance approaches 3

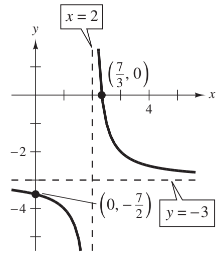
101. Proof

103. False. Let $f(x) = \frac{2x}{\sqrt{x^2+2}}$. $f'(x) > 0$ for all real numbers.

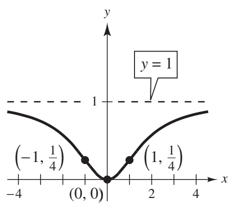
Section 3.6 *(page 212)*

1. d **2.** c **3.** a **4.** b

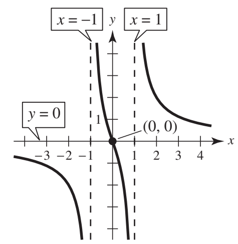
5.



7.



9.



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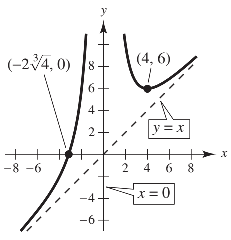
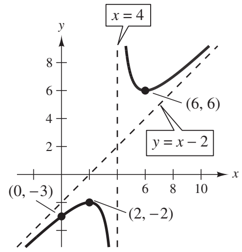
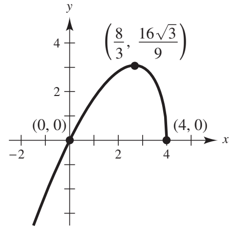


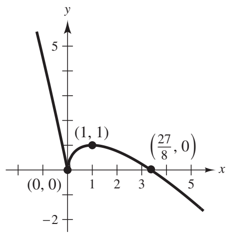
13.



15.



17.



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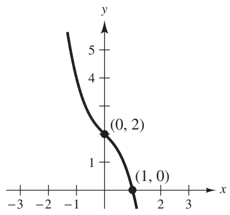
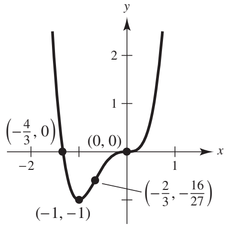
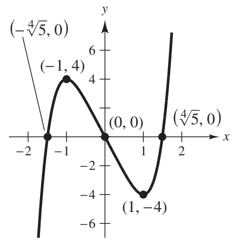


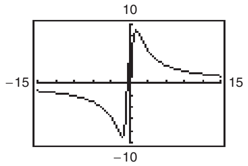
21.



23.

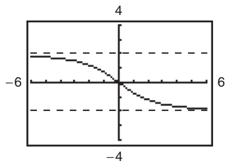


25.



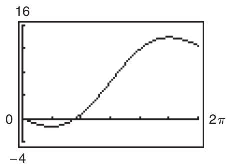
Minimum: $(-1.10, -9.05)$;
Maximum: $(1.10, 9.05)$;
Points of inflection:
$(-1.84, -7.86)$, $(1.84, 7.86)$;
Vertical asymptote: $x = 0$;
Horizontal asymptote: $y = 0$

27.



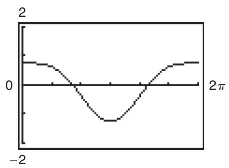
Point of inflection: $(0, 0)$;
Horizontal asymptotes: $y = \pm 2$

29.



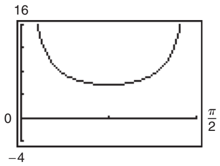
Relative minimum:
$\left(\frac{\pi}{3}, \frac{2\pi}{3} - 2\sqrt{3}\right)$;
Relative maximum:
$\left(\frac{5\pi}{3}, \frac{10\pi}{3} + 2\sqrt{3}\right)$;
Points of inflection: $(0, 0)$, $(\pi, 2\pi)$, $(2\pi, 4\pi)$

31.



Relative minimum: $\left(\pi, -\frac{5}{4}\right)$;
Points of inflection:
$\left(\frac{2\pi}{3}, -\frac{3}{8}\right)$, $\left(\frac{4\pi}{3}, -\frac{3}{8}\right)$

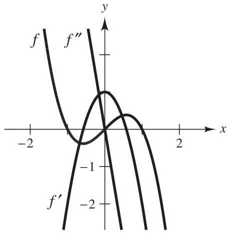
33.



Relative minimum: $\left(\frac{\pi}{4}, 4\sqrt{2}\right)$;
Vertical asymptotes: $x = 0, \frac{\pi}{2}$

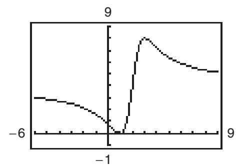
35. f is decreasing on $(2, 8)$, and therefore $f(3) > f(5)$.

37.



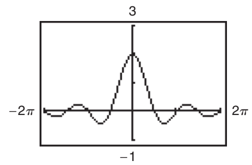
The zeros of f' correspond to the points where the graph of f has horizontal tangents. The zero of f'' corresponds to the point where the graph of f' has a horizontal tangent.

39.



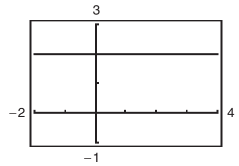
The graph crosses the horizontal asymptote $y = 4$. The graph of a function f does not cross its vertical asymptote $x = c$ because $f(c)$ does not exist.

41.



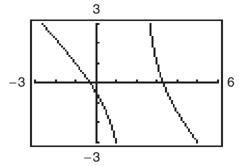
The graph has a hole at $x = 0$. The graph crosses the horizontal asymptote $y = 0$. The graph of a function f does not cross its vertical asymptote $x = c$ because $f(c)$ does not exist.

43.



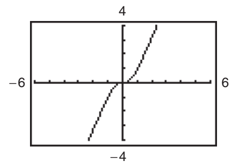
The graph has a hole at $x = 3$. The rational function is not reduced to lowest terms.

45.



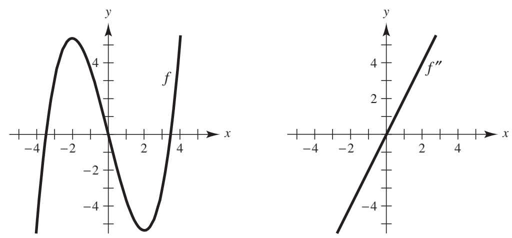
The graph appears to approach the line $y = -x + 1$, which is the slant asymptote.

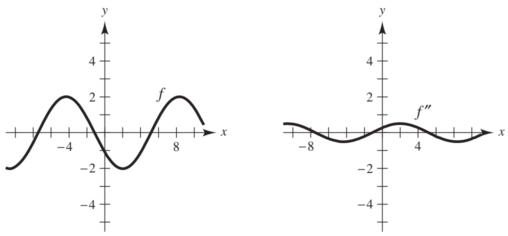
47.

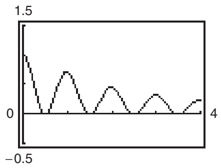


The graph appears to approach the line $y = 2x$, which is the slant asymptote.

49.



51.


53. (a)


The graph has holes at $x = 0$ and at $x = 4$.
Visually approximated critical numbers: $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

(b) $f'(x) = \dfrac{-x\cos^2(\pi x)}{(x^2+1)^{3/2}} - \dfrac{2\pi\sin(\pi x)\cos(\pi x)}{\sqrt{x^2+1}}$;

Approximate critical numbers: $\frac{1}{2}, 0.97, \frac{3}{2}, 1.98, \frac{5}{2}, 2.98, \frac{7}{2}$;
The critical numbers where maxima occur appear to be integers in part (a), but by approximating them using f', you can see that they are not integers.

55. Answers will vary. Sample answer: $y = 1/(x-3)$

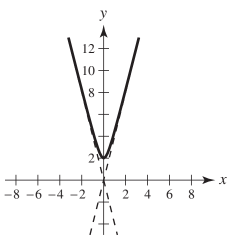
57. Answers will vary.
Sample answer: $y = (3x^2 - 7x - 5)/(x-3)$

59. (a) x_0, x_2, x_4 (b) x_2, x_3 (c) x_1 (d) x_1 (e) x_2, x_3

61. (a)–(h) Proofs

63. Answers will vary. Sample answer: The graph has a vertical asymptote at $x = b$. If a and b are both positive or both negative, then the graph of f approaches ∞ as x approaches b, and the graph has a minimum at $x = -b$. If a and b have opposite signs, then the graph of f approaches $-\infty$ as x approaches b, and the graph has a maximum at $x = -b$.

65. $y = 4x,\ y = -4x$



67. Putnam Problem 13(i), 1939

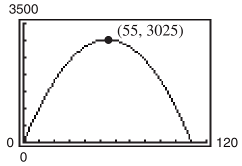
Section 3.7 *(page 220)*

1. (a) and (b)

First Number, x	Second Number	Product, P
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$
30	$110 - 30$	$30(110 - 30) = 2400$
40	$110 - 40$	$40(110 - 40) = 2800$
50	$110 - 50$	$50(110 - 50) = 3000$
60	$110 - 60$	$60(110 - 60) = 3000$
70	$110 - 70$	$70(110 - 70) = 2800$
80	$110 - 80$	$80(110 - 80) = 2400$
90	$110 - 90$	$90(110 - 90) = 1800$
100	$110 - 100$	$100(110 - 100) = 1000$

The maximum is attained near $x = 50$ and 60.

(c) $P = x(110 - x)$

(d)


(e) 55 and 55

3. $S/2$ and $S/2$ **5.** 21 and 7 **7.** 54 and 27

9. $l = w = 20$ m **11.** $l = w = 4\sqrt{2}$ ft **13.** $(1, 1)$

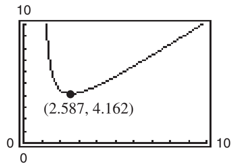
15. $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

17. Dimensions of page: $(2 + \sqrt{30})$ in. $\times$ $(2 + \sqrt{30})$ in.

19. 700×350 m

21. Rectangular portion: $16/(\pi + 4) \times 32/(\pi + 4)$ ft

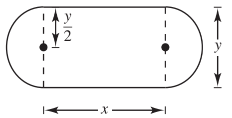
23. (a) $L = \sqrt{x^2 + 4 + \dfrac{8}{x-1} + \dfrac{4}{(x-1)^2}},\quad x > 1$

(b)


Minimum when $x \approx 2.587$

(c) $(0, 0), (2, 0), (0, 4)$

25. Width: $5\sqrt{2}/2$; Length: $5\sqrt{2}$

27. (a)


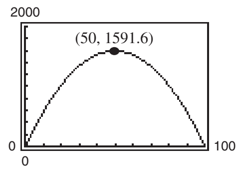
(b)

Length, x	Width, y	Area, xy
10	$2/\pi\,(100 - 10)$	$(10)(2/\pi)(100 - 10) \approx 573$
20	$2/\pi\,(100 - 20)$	$(20)(2/\pi)(100 - 20) \approx 1019$
30	$2/\pi\,(100 - 30)$	$(30)(2/\pi)(100 - 30) \approx 1337$
40	$2/\pi\,(100 - 40)$	$(40)(2/\pi)(100 - 40) \approx 1528$
50	$2/\pi\,(100 - 50)$	$(50)(2/\pi)(100 - 50) \approx 1592$
60	$2/\pi\,(100 - 60)$	$(60)(2/\pi)(100 - 60) \approx 1528$

The maximum area of the rectangle is approximately 1592 m^2.

(c) $A = 2/\pi(100x - x^2),\quad 0 < x < 100$

(d) $\dfrac{dA}{dx} = \dfrac{2}{\pi}(100 - 2x)$
$= 0$ when $x = 50$;
The maximum value is approximately 1592 when $x = 50$.

(e)


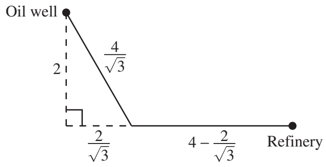
29. $18 \times 18 \times 36$ in.

31. No. The volume changes because the shape of the container changes when it is squeezed.

33. $r = \sqrt[3]{21/(2\pi)} \approx 1.50$ ($h = 0$, so the solid is a sphere.)

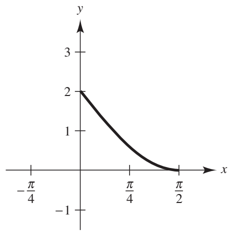
35. Side of square: $\dfrac{10\sqrt{3}}{9 + 4\sqrt{3}}$; Side of triangle: $\dfrac{30}{9 + 4\sqrt{3}}$

37. $w = \left(20\sqrt{3}\right)/3$ in., $h = \left(20\sqrt{6}\right)/3$ in.

39. 

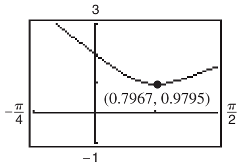
The path of the pipe should go underwater from the oil well to the coast following the hypotenuse of a right triangle with leg lengths of 2 miles and $2/\sqrt{3}$ miles for a distance of $4/\sqrt{3}$ miles. Then the pipe should go down the coast to the refinery for a distance of $\left(4 - 2/\sqrt{3}\right)$ miles.

41. One mile from the nearest point on the coast

43. 

(a) Origin to y-intercept: 2;
Origin to x-intercept: $\pi/2$

(b) $d = \sqrt{x^2 + (2 - 2\sin x)^2}$



(c) Minimum distance is 0.9795 when $x \approx 0.7967$.

45. About 1.153 radians or 66° **47.** 8%

49. $y = \frac{64}{141}x$; $S \approx 6.1$ mi **51.** $y = \frac{3}{10}x$; $S_3 \approx 4.50$ mi

53. Putnam Problem A1, 1986

Section 3.8 *(page 229)*

In the answers for Exercises 1 and 3, the values in the tables have been rounded for convenience. Because a calculator and a computer program calculates internally using more digits than they display, you may produce slightly different values from those shown in the tables.

1.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	2.2000	−0.1600	4.4000	−0.0364	2.2364
2	2.2364	0.0015	4.4728	0.0003	2.2361

3.

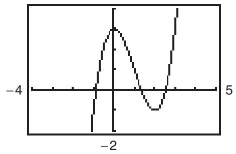
n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1.6	−0.0292	−0.9996	0.0292	1.5708
2	1.5708	0	−1	0	1.5708

5. −1.587 **7.** 0.682 **9.** 1.250, 5.000

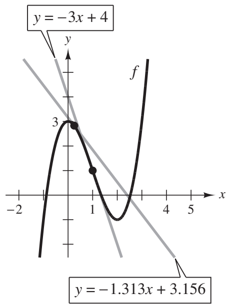
11. 0.900, 1.100, 1.900 **13.** 1.935 **15.** 0.569

17. 4.493 **19.** (a) Proof (b) $\sqrt{5} \approx 2.236$; $\sqrt{7} \approx 2.646$

21. $f'(x_1) = 0$ **23.** 0.74 **25.** Proof

27. (a) 

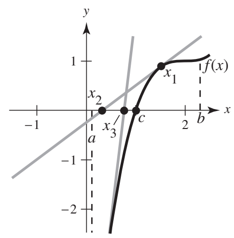
(b) 1.347 (c) 2.532

(d) 

x-intercept of $y = -3x + 4$ is $\frac{4}{3}$.
x-intercept of
$y = -1.313x + 3.156$
is approximately 2.404.

(e) If the initial estimate $x = x_1$ is not sufficiently close to the desired zero of a function, then the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

29. Answers will vary. Sample answer:
If f is a function continuous on $[a, b]$ and differentiable on (a, b), where $c \in [a, b]$ and $f(c) = 0$, then Newton's Method uses tangent lines to approximate c. First, estimate an initial x_1 close to c. (See graph.) Then determine x_2 using $x_2 = x_1 - f(x_1)/f'(x_1)$. Calculate a third estimate x_3 using $x_3 = x_2 - f(x_2)/f'(x_2)$. Continue this process until $|x_n - x_{n+1}|$ is within the desired accuracy, and let x_{n+1} be the final approximation of c.



31. (1.939, 0.240) **33.** $x \approx 1.563$ mi

35. False; let $f(x) = \dfrac{x^2 - 1}{x - 1}$. **37.** True **39.** 0.217

Section 3.9 *(page 236)*

1. $T(x) = 4x - 4$

x	1.9	1.99	2	2.01	2.1
$f(x)$	3.610	3.960	4	4.040	4.410
$T(x)$	3.600	3.960	4	4.040	4.400

3. $T(x) = 80x - 128$

x	1.9	1.99	2	2.01	2.1
$f(x)$	24.761	31.208	32	32.808	40.841
$T(x)$	24.000	31.200	32	32.800	40.000

5. $T(x) = (\cos 2)(x - 2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x)$	0.946	0.913	0.909	0.905	0.863
$T(x)$	0.951	0.913	0.909	0.905	0.868

7. $\Delta y = 0.331$; $dy = 0.3$ **9.** $\Delta y = -0.039$; $dy = -0.040$

11. $6x\,dx$ **13.** $(x \sec^2 x + \tan x)\,dx$

15. $-\dfrac{13}{(2x - 1)^2}\,dx$ **17.** $\dfrac{-x}{\sqrt{9 - x^2}}\,dx$ **19.** $(3 - \sin 2x)\,dx$

21. (a) 0.9 (b) 1.04 **23.** (a) 8.035 (b) 7.95

25. (a) $\pm\frac{5}{8}$ in.^{2} (b) 0.625%

27. (a) ± 10.75 cm^2 (b) about 1.19%

29. (a) ± 20.25 in.^{3} (b) ± 5.4 in.^{2} (c) 0.6%; 0.4%

31. 27.5 mi; About 7.3% **33.** (a) $\frac{1}{4}$% (b) 216 sec = 3.6 min

35. 6407 ft

37. $f(x) = \sqrt{x},\ dy = \dfrac{1}{2\sqrt{x}}\,dx$

$f(99.4) \approx \sqrt{100} + \dfrac{1}{2\sqrt{100}}(-0.6) = 9.97$

Calculator: 9.97

39. $f(x) = \sqrt[4]{x},\ dy = \dfrac{1}{4x^{3/4}}\,dx$

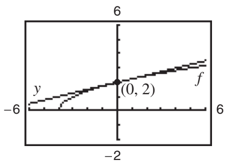
$f(624) \approx \sqrt[4]{625} + \dfrac{1}{4(625)^{3/4}}(-1) = 4.998$

Calculator: 4.998

41. $y - f(0) = f'(0)(x - 0)$

$y - 2 = \frac{1}{4}x$

$y = 2 + x/4$



43. The value of dy becomes closer to the value of Δy as Δx decreases.

45. $f(x) = \sqrt{x};\ dy = \dfrac{1}{2\sqrt{x}}\,dx$

$f(4.02) \approx \sqrt{4} + \dfrac{1}{2\sqrt{4}}(0.02) = 2 + \dfrac{1}{4}(0.02)$

47. True **49.** True

Review Exercises for Chapter 3 *(page 238)*

1. Maximum: (0, 0); Minimum: $\left(-\frac{5}{2}, -\frac{25}{4}\right)$

3. Maximum: (4, 0); Minimum: (0, −2)

5. Maximum: $\left(3, \frac{2}{3}\right)$; Minimum: $\left(-3, -\frac{2}{3}\right)$

7. Maximum: (2π, 17.57); Minimum: (2.73, 0.88)

9. $f(0) \neq f(4)$ **11.** Not continuous on $[-2, 2]$

13. $f'\left(\dfrac{2744}{729}\right) = \dfrac{3}{7}$ **15.** f is not differentiable at $x = 5$.

17. $f'(0) = 1$

19. No; The function has a discontinuity at $x = 0$, which is in the interval $[-2, 1]$.

21. Increasing on $\left(-\frac{3}{2}, \infty\right)$; Decreasing on $\left(-\infty, -\frac{3}{2}\right)$

23. Increasing on $(-\infty, 1)$, $\left(\frac{7}{3}, \infty\right)$; Decreasing on $\left(1, \frac{7}{3}\right)$

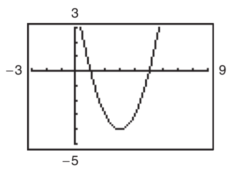
25. Increasing on $(1, \infty)$; Decreasing on $(0, 1)$

27. (a) Critical number: $x = 3$

(b) Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$

(c) Relative minimum: (3, −4)

(d)

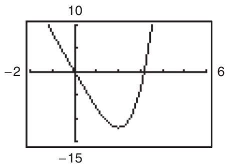


29. (a) Critical number: $t = 2$

(b) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$

(c) Relative minimum: (2, −12)

(d)



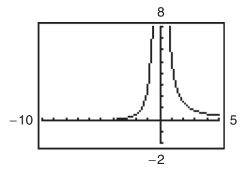
31. (a) Critical number: $x = -8$; Discontinuity: $x = 0$

(b) Increasing on $(-8, 0)$;

Decreasing on $(-\infty, -8)$ and $(0, \infty)$

(c) Relative minimum: $\left(-8, -\frac{1}{16}\right)$

(d)



33. (a) Critical numbers: $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

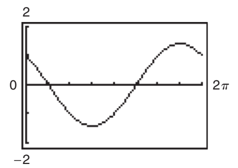
(b) Increasing on $\left(\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right)$;

Decreasing on $\left(0, \dfrac{3\pi}{4}\right)$ and $\left(\dfrac{7\pi}{4}, 2\pi\right)$

(c) Relative minimum: $\left(\dfrac{3\pi}{4}, -\sqrt{2}\right)$;

Relative maximum: $\left(\dfrac{7\pi}{4}, \sqrt{2}\right)$

(d)



35. (3, −54); Concave upward: $(3, \infty)$; Concave downward: $(-\infty, 3)$

37. No point of inflection; Concave upward: $(-5, \infty)$

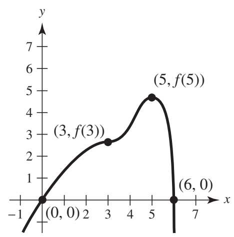
39. $(\pi/2, \pi/2)$, $(3\pi/2, 3\pi/2)$; Concave upward: $(\pi/2, 3\pi/2)$; Concave downward: $(0, \pi/2)$, $(3\pi/2, 2\pi)$

41. Relative minimum: (−9, 0)

43. Relative maxima: $\left(\sqrt{2}/2, 1/2\right)$, $\left(-\sqrt{2}/2, 1/2\right)$; Relative minimum: (0, 0)

45. Relative maximum: (−3, −12); Relative minimum: (3, 12)

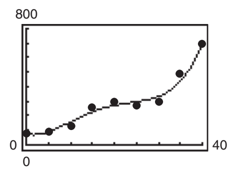
47.



49. Increasing and concave down

51. (a) $D = 0.00188t^4 - 0.1273t^2 + 2.672t^2 - 7.81t + 77.1$

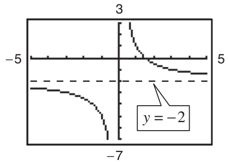
(b)



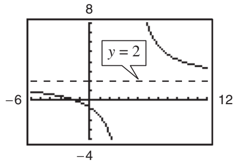
(c) Maximum in 2010; Minimum in 1970 (d) 2010

53. 8 **55.** $\frac{2}{3}$ **57.** $-\infty$ **59.** 0 **61.** 6

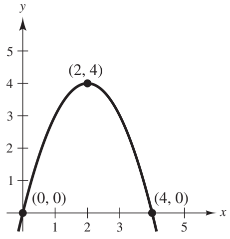
63.



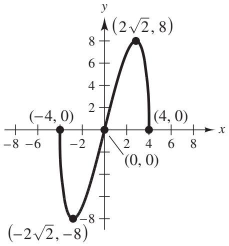
65.



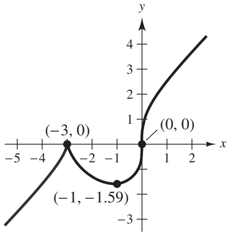
67.



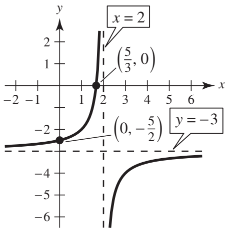
69.



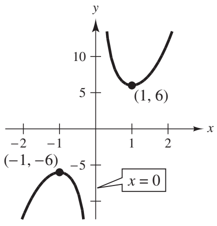
71.



73.



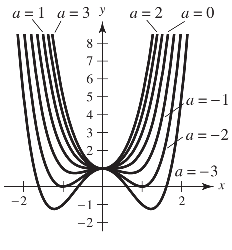
75.



77. $x = 50$ ft and $y = \frac{200}{3}$ ft **79.** (0, 0), (5, 0), (0, 10)
81. 14.05 ft **83.** $32\pi r^3/81$ **85.** $-1.532, -0.347, 1.879$
87. $-2.182, -0.795$ **89.** -0.755
91. $\Delta y = 0.03005$; $dy = 0.03$
93. $dy = (1 - \cos x + x \sin x)\, dx$ **95.** (a) $\pm 8.1\pi$ cm^3
(b) $\pm 1.8\pi$ cm^2 (c) About 0.83%; About 0.56%

P.S. Problem Solving *(page 241)*

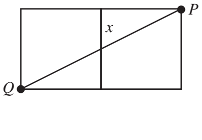
1. Choices of a may vary.



(a) One relative minimum at (0, 1) for $a \geq 0$
(b) One relative maximum at (0, 1) for $a < 0$
(c) Two relative minima for $a < 0$ when $x = \pm\sqrt{-a/2}$
(d) If $a < 0$, then there are three critical points; if $a \geq 0$, then there is only one critical point.

3. All c, where c is a real number **5.** Proof

7. The bug should head towards the midpoint of the opposite side. Without calculus, imagine opening up the cube. The shortest distance is the line PQ, passing through the midpoint as shown.

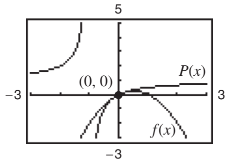


9. $a = 6, b = 1, c = 2$ **11.** Proof

13. Greatest slope: $\left(-\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$; Least slope: $\left(\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$

15. Proof **17.** Proof; Point of inflection: (1, 0)

19. (a) $P(x) = x - x^2$
(b)



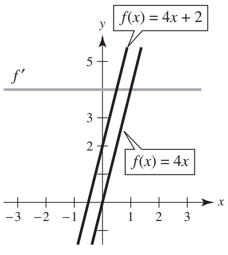
Chapter 4

Section 4.1 *(page 251)*

1. Proof **3.** $y = 3t^3 + C$ **5.** $y = \frac{2}{5}x^{5/2} + C$

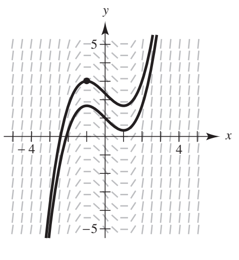
	Original Integral	Rewrite	Integrate	Simplify
7.	$\int \sqrt[3]{x}\, dx$	$\int x^{1/3}\, dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
9.	$\int \frac{1}{x\sqrt{x}}\, dx$	$\int x^{-3/2}\, dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$

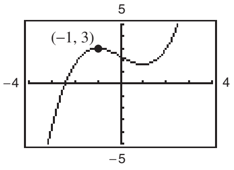
11. $\frac{1}{2}x^2 + 7x + C$ **13.** $\frac{1}{6}x^6 + x + C$
15. $\frac{2}{5}x^{5/2} + x^2 + x + C$ **17.** $\frac{3}{5}x^{5/3} + C$
19. $-1/(4x^4) + C$ **21.** $\frac{2}{3}x^{3/2} + 12x^{1/2} + C$
23. $x^3 + \frac{1}{2}x^2 - 2x + C$ **25.** $5 \sin x - 4 \cos x + C$
27. $t + \csc t + C$ **29.** $\tan\theta + \cos\theta + C$ **31.** $\tan y + C$
33. Answers will vary. Sample answer:



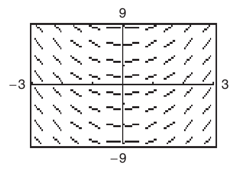
35. $f(x) = 3x^2 + 8$ **37.** $h(t) = 2t^4 + 5t - 11$
39. $f(x) = x^2 + x + 4$ **41.** $f(x) = -4\sqrt{x} + 3x$

43. (a) Answers will vary. (b) $y = \frac{x^3}{3} - x + \frac{7}{3}$
Sample answer:

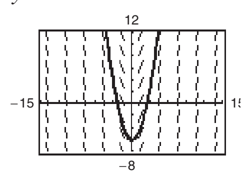




45. (a)

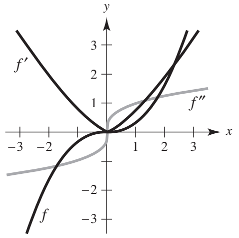


(b) $y = x^2 - 6$
(c)



47. When you evaluate the integral $\int f(x)\,dx$, you are finding a function $F(x)$ that is an antiderivative of $f(x)$. So, there is no difference.

49.



51. (a) $h(t) = \frac{3}{4}t^2 + 5t + 12$ (b) 69 cm **53.** 62.25 ft

55. (a) $t \approx 2.562$ sec (b) $v(t) \approx -65.970$ ft/sec

57. $v_0 \approx 62.3$ m/sec **59.** 320 m; -32 m/sec

61. (a) $v(t) = 3t^2 - 12t + 9$; $a(t) = 6t - 12$
(b) (0, 1), (3, 5) (c) -3

63. $a(t) = -1/(2t^{3/2})$; $x(t) = 2\sqrt{t} + 2$

65. (a) 1.18 m/sec^2 (b) 190 m

67. (a) 300 ft (b) 60 ft/sec $\approx$ 41 mi/h

69. False. f has an infinite number of antiderivatives, each differing by a constant.

71. True **73.** True **75.** $f(x) = \dfrac{x^3}{3} - 4x + \dfrac{16}{3}$

77. Proof

Section 4.2 *(page 263)*

1. 75 **3.** $\dfrac{158}{85}$ **5.** $4c$ **7.** $\displaystyle\sum_{i=1}^{11} \frac{1}{5i}$

9. $\displaystyle\sum_{j=1}^{6} \left[7\left(\frac{j}{6}\right) + 5\right]$ **11.** $\displaystyle\frac{2}{n}\sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^3 - \left(\frac{2i}{n}\right)\right]$ **13.** 84

15. 1200 **17.** 2470 **19.** 12,040

21. $(n + 2)/n$
$n = 10$: $S = 1.2$
$n = 100$: $S = 1.02$
$n = 1000$: $S = 1.002$
$n = 10{,}000$: $S = 1.0002$

23. $[2(n + 1)(n - 1)]/n^2$
$n = 10$: $S = 1.98$
$n = 100$: $S = 1.9998$
$n = 1000$: $S = 1.999998$
$n = 10{,}000$: $S = 1.99999998$

25. $13 <$ (Area of region) < 15

27. $55 <$ (Area of region) < 74.5

29. $0.7908 <$ (Area of region) < 1.1835

31. The area of the shaded region falls between 12.5 square units and 16.5 square units.

33. $A \approx S \approx 0.768$
$A \approx s \approx 0.518$

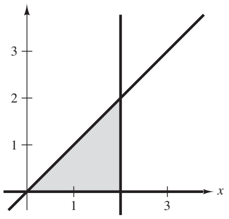
35. $A \approx S \approx 0.746$
$A \approx s \approx 0.646$

37. $\displaystyle\lim_{n\to\infty} \left[\frac{12(n + 1)}{n}\right] = 12$

39. $\displaystyle\lim_{n\to\infty} \frac{1}{6}\left(\frac{2n^3 - 3n^2 + n}{n^3}\right) = \frac{1}{3}$

41. $\displaystyle\lim_{n\to\infty} [(3n + 1)/n] = 3$

43. (a)



(b) $\Delta x = (2 - 0)/n = 2/n$

(c) $\displaystyle s(n) = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x = \sum_{i=1}^{n} [(i - 1)(2/n)](2/n)$

(d) $\displaystyle S(n) = \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n} [i(2/n)](2/n)$

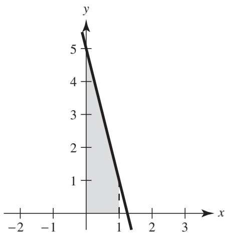
(e)

n	5	10	50	100
$s(n)$	1.6	1.8	1.96	1.98
$S(n)$	2.4	2.2	2.04	2.02

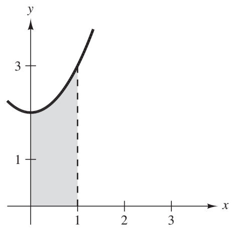
(f) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} [(i - 1)(2/n)](2/n) = 2$;

$\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} [i(2/n)](2/n) = 2$

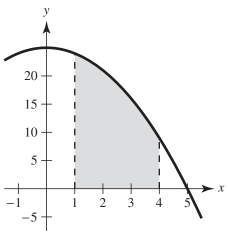
45. $A = 3$



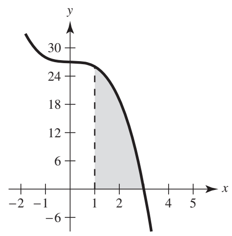
47. $A = \frac{7}{3}$



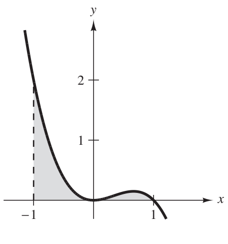
49. $A = 54$



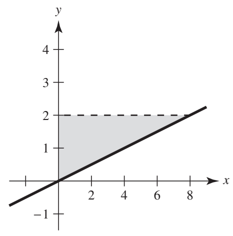
51. $A = 34$



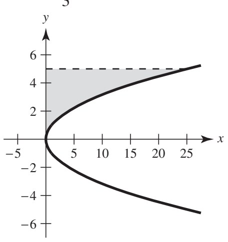
53. $A = \frac{2}{3}$



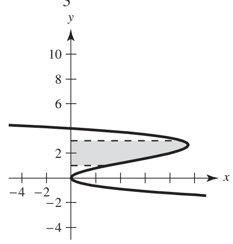
55. $A = 8$



57. $A = \frac{125}{3}$

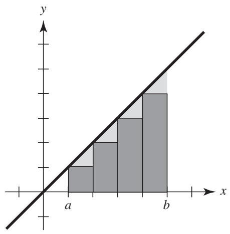


59. $A = \frac{44}{3}$

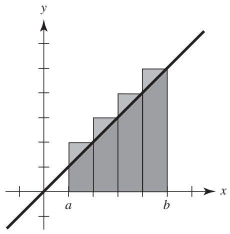


61. $\frac{69}{8}$ **63.** 0.345 **65.** b

67. You can use the line $y = x$ bounded by $x = a$ and $x = b$. The sum of the areas of the inscribed rectangles in the figure below is the lower sum.

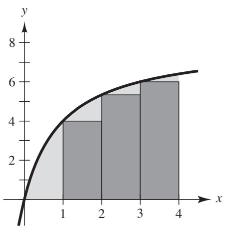


The sum of the areas of the circumscribed rectangles in the figure below is the upper sum.



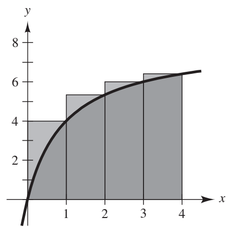
The rectangles in the first graph do not contain all of the area of the region, and the rectangles in the second graph cover more than the area of the region. The exact value of the area lies between these two sums.

69. (a)



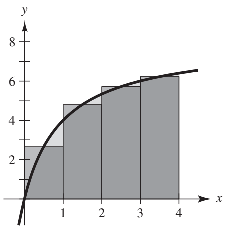
$s(4) = \frac{46}{3}$

(b)



$S(4) = \frac{326}{15}$

(c)



$M(4) = \frac{6112}{315}$

(d) Proof

(e)

n	4	8	20	100	200
$s(n)$	15.333	17.368	18.459	18.995	19.060
$S(n)$	21.733	20.568	19.739	19.251	19.188
$M(n)$	19.403	19.201	19.137	19.125	19.125

(f) Because f is an increasing function, $s(n)$ is always increasing and $S(n)$ is always decreasing.

71. True

73. Suppose there are n rows and $n + 1$ columns. The stars on the left total $1 + 2 + \cdots + n$, as do the stars on the right. There are $n(n + 1)$ stars in total. So, $2[1 + 2 + \cdots + n] = n(n + 1)$ and $1 + 2 + \cdots + n = [n(n + 1)]/2$.

75. For n odd, $\left(\frac{n+1}{2}\right)^2$ blocks;

For n even, $\frac{n^2 + 2n}{4}$ blocks

77. Putnam Problem B1, 1989

Section 4.3 *(page 273)*

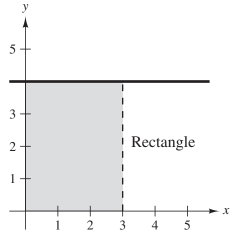
1. $2\sqrt{3} \approx 3.464$ **3.** 32 **5.** 0 **7.** $\frac{10}{3}$

9. $\int_{-1}^{5} (3x + 10)\,dx$ **11.** $\int_{0}^{3} \sqrt{x^2 + 4}\,dx$ **13.** $\int_{0}^{4} 5\,dx$

15. $\int_{-4}^{4} (4 - |x|)\,dx$ **17.** $\int_{-5}^{5} (25 - x^2)\,dx$

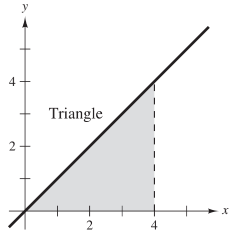
19. $\int_{0}^{\pi/2} \cos x\,dx$ **21.** $\int_{0}^{2} y^3\,dy$

23.



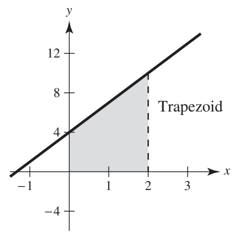
$A = 12$

25.



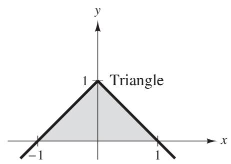
$A = 8$

27.



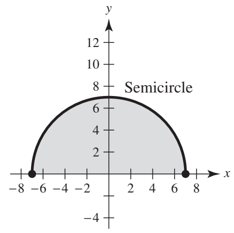
$A = 14$

29.



$A = 1$

31.



$A = 49\pi/2$

33. -6 **35.** 48 **37.** -12

39. 16 **41.** (a) 13 (b) -10 (c) 0 (d) 30

43. (a) 8 (b) -12 (c) -4 (d) 30 **45.** $-48, 88$

47. (a) $-\pi$ (b) 4 (c) $-(1 + 2\pi)$ (d) $3 - 2\pi$ (e) $5 + 2\pi$ (f) $23 - 2\pi$

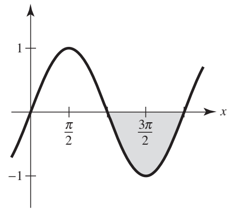
49. (a) 14 (b) 4 (c) 8 (d) 0 **51.** 40 **53.** a **55.** d

57. No. There is a discontinuity at $x = 4$.

59. $a = -2,\ b = 5$

61. Answers will vary. Sample answer: $a = \pi,\ b = 2\pi$

$\int_{\pi}^{2\pi} \sin x\,dx < 0$



63. True **65.** True **67.** False. $\int_{0}^{2} (-x)\,dx = -2$

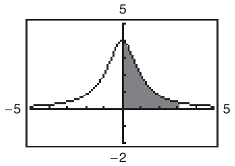
69. 272 **71.** Proof

73. No. No matter how small the subintervals, the number of both rational and irrational numbers within each subinterval is infinite, and $f(c_i) = 0$ or $f(c_i) = 1$.

75. $a = -1$ and $b = 1$ maximize the integral. **77.** $\frac{1}{3}$

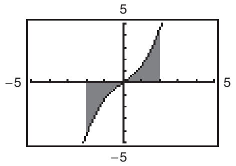
Section 4.4 *(page 288)*

1.



Positive

3.



Zero

5. 12 **7.** -2 **9.** $-\frac{10}{3}$ **11.** $\frac{1}{3}$ **13.** $\frac{1}{2}$ **15.** $\frac{2}{3}$

17. -4 **19.** $-\frac{1}{18}$ **21.** $-\frac{27}{20}$ **23.** $\frac{25}{2}$ **25.** $\frac{64}{3}$

27. $\pi + 2$ **29.** $\pi/4$ **31.** $2\sqrt{3}/3$ **33.** 0 **35.** $\frac{1}{6}$

37. 1 **39.** $\frac{52}{3}$ **41.** 20 **43.** $\frac{32}{3}$

45. $3\sqrt[3]{2}/2 \approx 1.8899$ **47.** $2\sqrt{3} \approx 3.4641$

49. $\pm\arccos\sqrt{\pi}/2 \approx \pm 0.4817$

51. Average value $= 6$
$x = \pm\sqrt{3} \approx \pm 1.7321$

53. Average value $= \frac{1}{4}$
$x = \sqrt[3]{2}/2 \approx 0.6300$

55. Average value $= 2/\pi$
$x \approx 0.690, x \approx 2.451$

57. About 540 ft

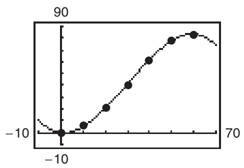
59. (a) 8 (b) $\frac{4}{3}$ (c) $\int_1^7 f(x)\,dx = 20$; Average value $= \frac{10}{3}$

61. (a) $F(x) = 500\sec^2 x$ (b) $1500\sqrt{3}/\pi \approx 827$ N

63. About 0.5318 L

65. (a) $v = -0.00086t^3 + 0.0782t^2 - 0.208t + 0.10$

(b)



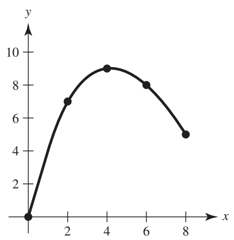
(c) 2475.6 m

67. $F(x) = 2x^2 - 7x$
$F(2) = -6$
$F(5) = 15$
$F(8) = 72$

69. $F(x) = -20/x + 20$
$F(2) = 10$
$F(5) = 16$
$F(8) = \frac{35}{2}$

71. $F(x) = \sin x - \sin 1$
$F(2) = \sin 2 - \sin 1 \approx 0.0678$
$F(5) = \sin 5 - \sin 1 \approx -1.8004$
$F(8) = \sin 8 - \sin 1 \approx 0.1479$

73. (a) $g(0) = 0, g(2) \approx 7, g(4) \approx 9, g(6) \approx 8, g(8) \approx 5$
(b) Increasing: $(0, 4)$; Decreasing: $(4, 8)$
(c) A maximum occurs at $x = 4$.
(d)

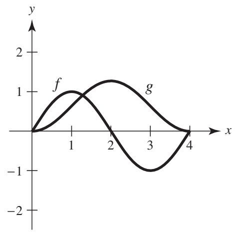


75. $\frac{1}{2}x^2 + 2x$ **77.** $\frac{3}{4}x^{4/3} - 12$ **79.** $\tan x - 1$

81. $x^2 - 2x$ **83.** $\sqrt{x^4 + 1}$ **85.** $x\cos x$ **87.** 8

89. $\cos x\sqrt{\sin x}$ **91.** $3x^2\sin x^6$

93.



An extremum of g occurs at $x = 2$.

95. (a) $\frac{3}{2}$ ft to the right
(b) $\frac{113}{10}$ ft

97. (a) 0 ft (b) $\frac{63}{2}$ ft **99.** (a) 2 ft to the right (b) 2 ft

101. 28 units **103.** 8190 L

105. $f(x) = x^{-2}$ has a nonremovable discontinuity at $x = 0$.

107. $f(x) = \sec^2 x$ has a nonremovable discontinuity at $x = \pi/2$.

109. $2/\pi \approx 63.7\%$ **111.** True

113. $f'(x) = \dfrac{1}{(1/x)^2 + 1}\left(-\dfrac{1}{x^2}\right) + \dfrac{1}{x^2 + 1} = 0$

Because $f'(x) = 0$, $f(x)$ is constant.

115. (a) 0 (b) 0 (c) $xf(x) + \int_0^x f(t)\,dt$ (d) 0

Section 4.5 *(page 301)*

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (8x^2 + 1)^2(16x)\,dx$	$8x^2 + 1$	$16x\,dx$
3. $\int \tan^2 x\sec^2 x\,dx$	$\tan x$	$\sec^2 x\,dx$

5. $\frac{1}{5}(1 + 6x)^5 + C$ **7.** $\frac{2}{3}(25 - x^2)^{3/2} + C$

9. $\frac{1}{12}(x^4 + 3)^3 + C$ **11.** $\frac{1}{15}(x^3 - 1)^5 + C$

13. $\frac{1}{3}(t^2 + 2)^{3/2} + C$ **15.** $-\frac{15}{8}(1 - x^2)^{4/3} + C$

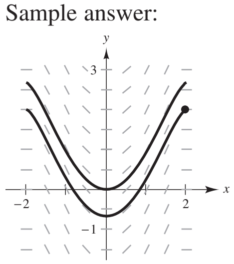
17. $1/[4(1 - x^2)^2] + C$ **19.** $-1/[3(1 + x^3)] + C$

21. $-\sqrt{1 - x^2} + C$ **23.** $-\frac{1}{4}(1 + 1/t)^4 + C$

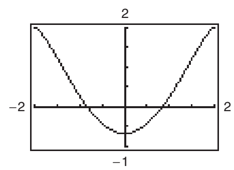
25. $\sqrt{2x} + C$ **27.** $2x^2 - 4\sqrt{16 - x^2} + C$

29. $-1/[2(x^2 + 2x - 3)] + C$

31. (a) Answers will vary.
Sample answer:



(b) $y = -\frac{1}{3}(4 - x^2)^{3/2} + 2$



33. $-\cos(\pi x) + C$

35. $\int \cos 8x\,dx = \dfrac{1}{8}\int (\cos 8x)(8)\,dx = \dfrac{1}{8}\sin 8x + C$

37. $-\sin(1/\theta) + C$

39. $\frac{1}{4}\sin^2 2x + C$ or $-\frac{1}{4}\cos^2 2x + C_1$ or $-\frac{1}{8}\cos 4x + C_2$

41. $\frac{1}{2}\tan^2 x + C$ or $\frac{1}{2}\sec^2 x + C_1$ **43.** $f(x) = 2\cos(x/2) + 4$

45. $f(x) = \frac{1}{12}(4x^2 - 10)^3 - 8$

47. $\frac{2}{5}(x + 6)^{5/2} - 4(x + 6)^{3/2} + C = \frac{2}{5}(x + 6)^{3/2}(x - 4) + C$

49. $-\left[\frac{2}{3}(1 - x)^{3/2} - \frac{4}{5}(1 - x)^{5/2} + \frac{2}{7}(1 - x)^{7/2}\right] + C =$
$-\frac{2}{105}(1 - x)^{3/2}(15x^2 + 12x + 8) + C$

51. $\frac{1}{8}\left[\frac{2}{5}(2x-1)^{5/2}+\frac{4}{3}(2x-1)^{3/2}-6(2x-1)^{1/2}\right]+C=$ $\left(\sqrt{2x-1}/15\right)(3x^2+2x-13)+C$

53. $-x-1-2\sqrt{x+1}+C$ or $-\left(x+2\sqrt{x+1}\right)+C_1$

55. 0 **57.** $12-\frac{8}{9}\sqrt{2}$ **59.** 2 **61.** $\frac{1}{2}$

63. $f(x)=(2x^3+1)^3+3$ **65.** 1209/28 **67.** $2\left(\sqrt{3}-1\right)$

69. $\frac{272}{15}$ **71.** $\frac{2}{3}$ **73.** (a) $\frac{64}{3}$ (b) $\frac{128}{3}$ (c) $-\frac{64}{3}$ (d) 64

75. $2\int_0^3 (4x^2-6)\,dx=36$

77. If $u=5-x^2$, then $du=-2x\,dx$ and $\int x(5-x^2)^3\,dx=-\frac{1}{2}\int(5-x^2)^3(-2x)\,dx=-\frac{1}{2}\int u^3\,du.$

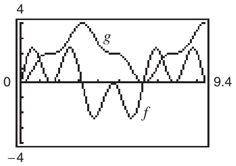
79. (a) $\int x^2\sqrt{x^3+1}\,dx$ (b) $\int \tan(3x)\sec^2(3x)\,dx$

81. \$340,000

83. (a) 102.532 thousand units (b) 102.352 thousand units (c) 74.5 thousand units

85. (a) $P_{0.50,\,0.75}\approx 35.3\%$ (b) $b\approx 58.6\%$

87. (a)

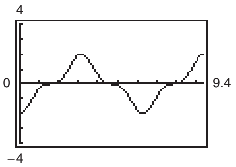


(b) g is nonnegative, because the graph of f is positive at the beginning and generally has more positive sections than negative ones.

(c) The points on g that correspond to the extrema of f are points of inflection of g.

(d) No, some zeros of f, such as $x=\pi/2$, do not correspond to extrema of g. The graph of g continues to increase after $x=\pi/2$, because f remains above the x-axis.

(e)



The graph of h is that of g shifted 2 units downward.

89. (a) and (b) Proofs

91. False. $\int(2x+1)^2\,dx=\frac{1}{6}(2x+1)^3+C$ **93.** True

95. True **97–99.** Proofs **101.** Putnam Problem A1, 1958

Section 4.6 *(page 310)*

	Trapezoidal	Simpson's	Exact
1.	2.7500	2.6667	2.6667
3.	4.2500	4.0000	4.0000
5.	20.2222	20.0000	20.0000
7.	12.6640	12.6667	12.6667
9.	0.3352	0.3334	0.3333

	Trapezoidal	Simpson's	Graphing Utility
11.	3.2833	3.2396	3.2413
13.	0.3415	0.3720	0.3927
15.	0.5495	0.5483	0.5493
17.	−0.0975	−0.0977	−0.0977
19.	0.1940	0.1860	0.1858

21. Trapezoidal: Linear (1st-degree) polynomials
Simpson's: Quadratic (2nd-degree) polynomials

23. (a) 1.500 (b) 0.000 **25.** (a) $\frac{1}{4}$ (b) $\frac{1}{12}$

27. (a) $n=366$ (b) $n=26$ **29.** (a) $n=77$ (b) $n=8$

31. (a) $n=130$ (b) $n=12$ **33.** (a) $n=643$ (b) $n=48$

35. (a) 24.5 (b) 25.67 **37.** 0.701 **39.** 89,250 m^2

41. 10,233.58 ft-lb **43.** 3.1416 **45.** 2.477 **47.** Proof

Review Exercises for Chapter 4 *(page 312)*

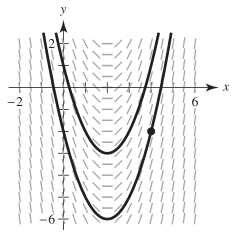
1. $\frac{x^2}{2}-6x+C$ **3.** $\frac{4}{3}x^3+\frac{1}{2}x^2+3x+C$

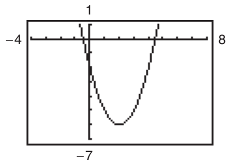
5. $x^2/2-4/x^2+C$ **7.** $x^2+9\cos x+C$

9. $y=1-3x^2$ **11.** $f(x)=4x^3-5x-3$

13. (a) Answers will vary. (b) $y=x^2-4x-2$

Sample answer:





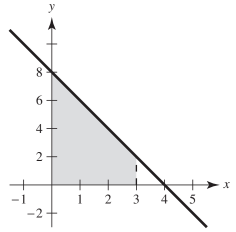
15. (a) 3 sec; 144 ft (b) $\frac{3}{2}$ sec (c) 108 ft

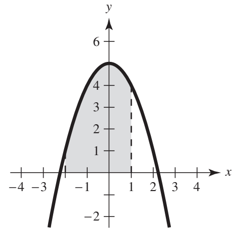
17. 240 ft/sec **19.** 60 **21.** $\sum_{n=1}^{10}\frac{1}{3n}$ **23.** 192

25. 420 **27.** 3310

29. 9.038 < (Area of region) < 13.038

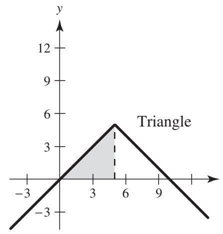
31. $A=15$ **33.** $A=12$





35. $\frac{27}{2}$ **37.** $\int_{-4}^{0}(2x+8)\,dx$

39.



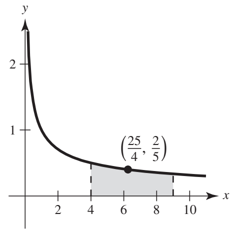
$A=\frac{25}{2}$

41. (a) 17 (b) 7 (c) 9 (d) 84

43. 56 **45.** 0 **47.** $\frac{422}{5}$ **49.** $\left(\sqrt{2}+2\right)/2$

51. $-\cos 2+1\approx 1.416$ **53.** 30 **55.** $\frac{1}{4}$

57. Average value $=\frac{2}{5}$, $x=\frac{25}{4}$



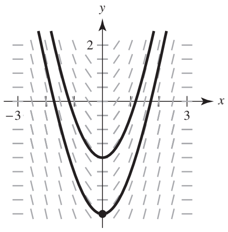
59. $x^2\sqrt{1+x^3}$ **61.** x^2+3x+2 **63.** $\frac{2}{3}\sqrt{x^3+3}+C$

65. $-\frac{1}{30}(1-3x^2)^5+C=\frac{1}{30}(3x^2-1)^5+C$

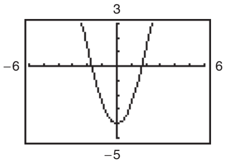
67. $\frac{1}{4}\sin^4 x + C$ **69.** $-2\sqrt{1 - \sin\theta} + C$

71. $\dfrac{1}{3\pi}(1 + \sec \pi x)^3 + C$

73. (a) Answers will vary. Sample answer:



(b) $y = -\frac{1}{3}(9 - x^2)^{3/2} + 5$



75. $\frac{455}{2}$ **77.** 2 **79.** $28\pi/15$ **81.** 2 **83.** $\frac{468}{7}$

85. (a) $\frac{64}{5}$ (b) $\frac{32}{5}$ (c) $\frac{96}{5}$ (d) -32

87. Trapezoidal Rule: 0.285
Simpson's Rule: 0.284
Graphing Utility: 0.284

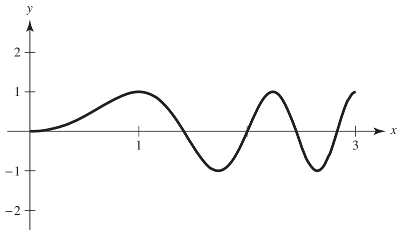
89. Trapezoidal Rule: 0.637
Simpson's Rule: 0.685
Graphing Utility: 0.704

P.S. Problem Solving *(page 315)*

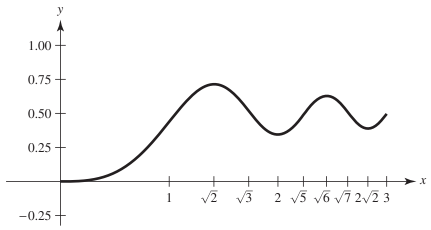
1. (a) $L(1) = 0$ (b) $L'(x) = 1/x$, $L'(1) = 1$
(c) $x \approx 2.718$ (d) Proof

3. (a) $\displaystyle\lim_{n\to\infty}\left[\frac{32}{n^5}\sum_{i=1}^{n} i^4 - \frac{64}{n^4}\sum_{i=1}^{n} i^3 + \frac{32}{n^3}\sum_{i=1}^{n} i^2\right]$
(b) $(16n^4 - 16)/(15n^4)$ (c) $16/15$

5. (a)

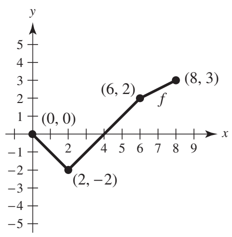


(b)



(c) Relative maxima at $x = \sqrt{2}, \sqrt{6}$
Relative minima at $x = 2, 2\sqrt{2}$
(d) Points of inflection at $x = 1, \sqrt{3}, \sqrt{5}, \sqrt{7}$

7. (a)



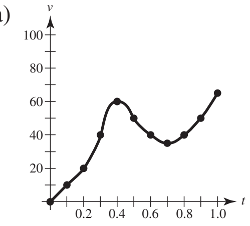
(b)

x	0	1	2	3	4	5	6	7	8
$F(x)$	0	$-\frac{1}{2}$	-2	$-\frac{7}{2}$	-4	$-\frac{7}{2}$	-2	$\frac{1}{4}$	3

(c) $x = 4, 8$ (d) $x = 2$

9. Proof **11.** $\frac{2}{3}$ **13.** $1 \le \displaystyle\int_0^1 \sqrt{1 + x^4}\,dx \le \sqrt{2}$

15. (a)



(b) (0, 0.4) and (0.7, 1.0) (c) 150 mi/h^2
(d) Total distance traveled in miles; 38.5 mi
(e) Sample answer: 100 mi/h^2

17. (a)–(c) Proofs

19. (a) $R(n), I, T(n), L(n)$
(b) $S(4) = \frac{1}{3}[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)] \approx 5.42$

Chapter 5

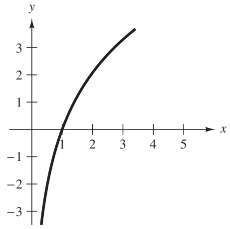
Section 5.1 *(page 325)*

1. (a) 3.8067 (b) $\ln 45 = \displaystyle\int_1^{45} \frac{1}{t}\,dt \approx 3.8067$

3. (a) -0.2231 (b) $\ln 0.8 = \displaystyle\int_1^{0.8} \frac{1}{t}\,dt \approx -0.2231$

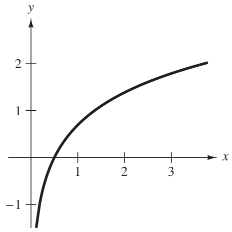
5. b **6.** d **7.** a **8.** c

9.



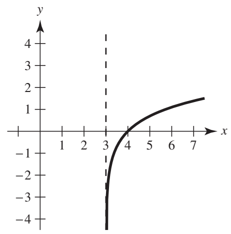
Domain: $x > 0$

11.



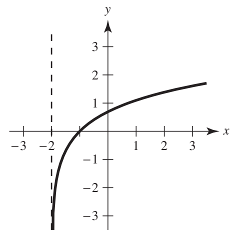
Domain: $x > 0$

13.



Domain: $x > 3$

15.



Domain: $x > -2$

17. (a) 1.7917 (b) -0.4055 (c) 4.3944 (d) 0.5493

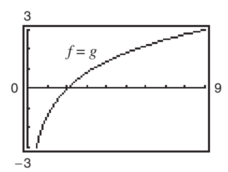
19. $\ln x - \ln 4$ **21.** $\ln x + \ln y - \ln z$

23. $\ln x + \frac{1}{2}\ln(x^2 + 5)$ **25.** $\frac{1}{2}[\ln(x - 1) - \ln x]$

27. $\ln z + 2\ln(z - 1)$ **29.** $\ln \dfrac{x - 2}{x + 2}$

31. $\ln \sqrt[3]{\dfrac{x(x + 3)^2}{x^2 - 1}}$ **33.** $\ln \dfrac{9}{\sqrt{x^2 + 1}}$

35. (a)



(b) $f(x) = \ln \dfrac{x^2}{4} = \ln x^2 - \ln 4$
$= 2\ln x - \ln 4$
$= g(x)$

37. $-\infty$ **39.** $\ln 4 \approx 1.3863$ **41.** $1/x$ **43.** $2/x$

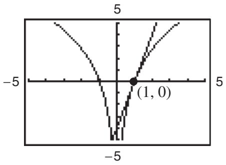
45. $4(\ln x)^3/x$ **47.** $2/(t+1)$ **49.** $\dfrac{2x^2-1}{x(x^2-1)}$

51. $\dfrac{1-x^2}{x(x^2+1)}$ **53.** $\dfrac{1-2\ln t}{t^3}$ **55.** $\dfrac{2}{x\ln x^2}=\dfrac{1}{x\ln x}$

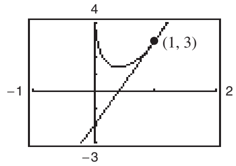
57. $\dfrac{1}{1-x^2}$ **59.** $\dfrac{-4}{x(x^2+4)}$ **61.** $\cot x$

63. $-\tan x+\dfrac{\sin x}{\cos x-1}$

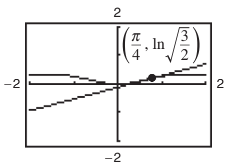
65. (a) $y=4x-4$
(b)



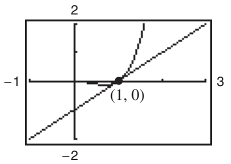
67. (a) $5x-y-2=0$
(b)



69. (a) $y=\frac{1}{3}x-\frac{1}{12}\pi+\frac{1}{2}\ln\left(\frac{3}{2}\right)$
(b)



71. (a) $y=x-1$
(b)



73. $\dfrac{2xy}{3-2y^2}$ **75.** $\dfrac{y(1-6x^2)}{1+y}$

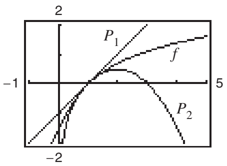
77. $xy''+y'=x(-2/x^2)+(2/x)=0$

79. Relative minimum: $\left(1,\frac{1}{2}\right)$

81. Relative minimum: $(e^{-1},-e^{-1})$

83. Relative minimum: (e,e); Point of inflection: $(e^2,e^2/2)$

85. $P_1(x)=x-1$; $P_2(x)=x-1-\frac{1}{2}(x-1)^2$



The values of f, P_1, and P_2 and their first derivatives agree at $x=1$.

87. $x\approx 0.567$ **89.** $(2x^2+1)/\sqrt{x^2+1}$

91. $\dfrac{3x^3+15x^2-8x}{2(x+1)^3\sqrt{3x-2}}$ **93.** $\dfrac{(2x^2+2x-1)\sqrt{x-1}}{(x+1)^{3/2}}$

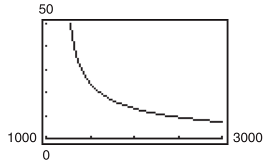
95. The domain of the natural logarithmic function is $(0,\infty)$, and the range is $(-\infty,\infty)$. The function is continuous, increasing, and one-to-one, and its graph is concave downward. In addition, if a and b are positive numbers and n is rational, then $\ln(1)=0$, $\ln(a\cdot b)=\ln a+\ln b$, $\ln(a^n)=n\ln a$, and $\ln(a/b)=\ln a-\ln b$.

97. (a) Yes. If the graph of g is increasing, then $g'(x)>0$. Because $f(x)>0$, you know that $f'(x)=g'(x)f(x)$ and thus $f'(x)>0$. Therefore, the graph of f is increasing.
(b) No. Let $f(x)=x^2+1$ (positive and concave up), and let $g(x)=\ln(x^2+1)$ (not concave up).

99. False. $\ln x+\ln 25=\ln 25x$

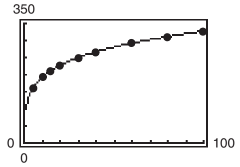
101. False. π is a constant, so $\dfrac{d}{dx}[\ln\pi]=0$.

103. (a)

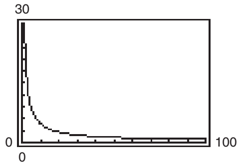


(b) 30 yr; \$503,434.80
(c) 20 yr; \$386,685.60
(d) When $x=1398.43$, $dt/dx\approx-0.0805$. When $x=1611.19$, $dt/dx\approx-0.0287$.
(e) Two benefits of a higher monthly payment are a shorter term and a lower total amount paid.

105. (a)



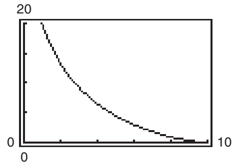
(c)



(b) $T'(10)\approx 4.75°/\text{lb/in.}^2$
$T'(70)\approx 0.97°/\text{lb/in.}^2$

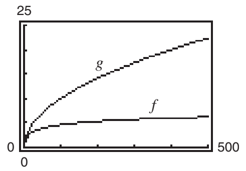
$\lim_{p\to\infty}T'(p)=0$
Answers will vary.

107. (a)



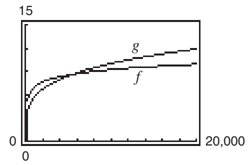
(b) When $x=5$, $dy/dx=-\sqrt{3}$. When $x=9$, $dy/dx=-\sqrt{19}/9$.
(c) $\lim_{x\to 10^-}\dfrac{dy}{dx}=0$

109. (a)



For $x>4$, $g'(x)>f'(x)$. g is increasing at a faster rate than f for large values of x.

(b)



For $x>256$, $g'(x)>f'(x)$. g is increasing at a faster rate than f for large values of x.

$f(x)=\ln x$ increases very slowly for large values of x.

Section 5.2 *(page 334)*

1. $5\ln|x|+C$ **3.** $\ln|x+1|+C$ **5.** $\frac{1}{2}\ln|2x+5|+C$

7. $\frac{1}{2}\ln|x^2-3|+C$ **9.** $\ln|x^4+3x|+C$

11. $x^2/2-\ln(x^4)+C$ **13.** $\frac{1}{3}\ln|x^3+3x^2+9x|+C$

15. $\frac{1}{2}x^2-4x+6\ln|x+1|+C$ **17.** $\frac{1}{3}x^3+5\ln|x-3|+C$

19. $\frac{1}{3}x^3-2x+\ln\sqrt{x^2+2}+C$ **21.** $\frac{1}{3}(\ln x)^3+C$

23. $-\frac{2}{3}\ln|1-3\sqrt{x}|+\text{C}$

25. $2\ln|x-1|-2/(x-1)+C$

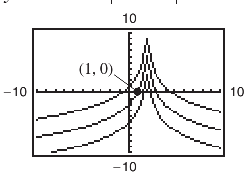
27. $\sqrt{2x}-\ln|1+\sqrt{2x}|+\text{C}$

29. $x+6\sqrt{x}+18\ln|\sqrt{x}-3|+C$ **31.** $3\ln\left|\sin\dfrac{\theta}{3}\right|+C$

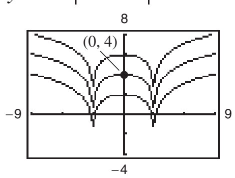
33. $-\frac{1}{2}\ln|\csc 2x+\cot 2x|+C$ **35.** $\frac{1}{3}\sin 3\theta-\theta+C$

37. $\ln|1+\sin t|+C$ **39.** $\ln|\sec x-1|+C$

41. $y=-3\ln|2-x|+C$

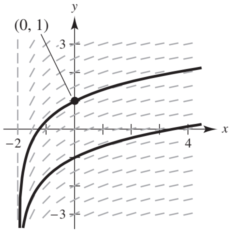


43. $y=\ln|x^2-9|+C$

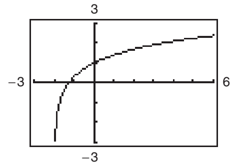


45. $f(x)=-2\ln x+3x-2$

47. (a)



(b) $y = \ln\left(\frac{x+2}{2}\right) + 1$



49. $\frac{5}{3}\ln 13 \approx 4.275$ **51.** $\frac{7}{3}$ **53.** $-\ln 3 \approx -1.099$

55. $\ln\left|\frac{2 - \sin 2}{1 - \sin 1}\right| \approx 1.929$ **57.** $2\left[\sqrt{x} - \ln\left(1 + \sqrt{x}\right)\right] + C$

59. $\ln\left(\frac{\sqrt{x} - 1}{\sqrt{x} + 1}\right) + 2\sqrt{x} + C$ **61.** $\ln\left(\sqrt{2} + 1\right) - \frac{\sqrt{2}}{2} \approx 0.174$

63. $1/x$ **65.** $1/x$ **67.** $6 \ln 3$ **69.** $\frac{1}{2}\ln 2$

71. $\frac{15}{2} + 8 \ln 2 \approx 13.045$ **73.** $(12/\pi)\ln\left(2 + \sqrt{3}\right) \approx 5.03$

75. Trapezoidal Rule: 20.2
Simpson's Rule: 19.4667

77. Trapezoidal Rule: 5.3368
Simpson's Rule: 5.3632

79. Power Rule **81.** Log Rule **83.** d **85.** $x = 2$

87. Proof

89. $-\ln|\cos x| + C = \ln|1/\cos x| + C = \ln|\sec x| + C$

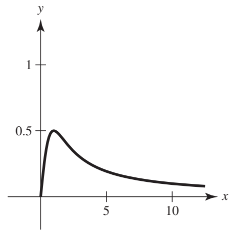
91. $\ln|\sec x + \tan x| + C = \ln\left|\frac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right| + C$
$= -\ln|\sec x - \tan x| + C$

93. 1 **95.** $1/(e - 1) \approx 0.582$

97. $P(t) = 1000(12 \ln|1 + 0.25t| + 1)$; $P(3) \approx 7715$

99. About 4.15 min

101.



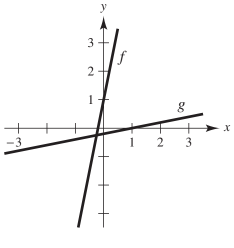
(a) $A = \frac{1}{2}\ln 2 - \frac{1}{4}$
(b) $0 < m < 1$
(c) $A = \frac{1}{2}(m - \ln m - 1)$

103. False. $\frac{1}{2}\ln x = \ln x^{1/2}$ **105.** True **107.** Proof

Section 5.3 *(page 343)*

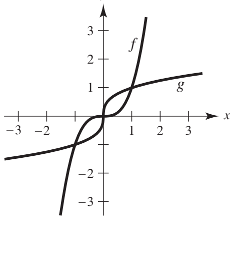
1. (a) $f(g(x)) = 5[(x - 1)/5] + 1 = x$;
$g(f(x)) = [(5x + 1) - 1]/5 = x$

(b)



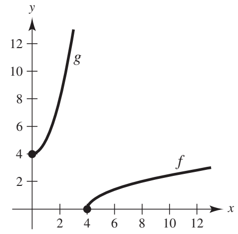
3. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$; $g(f(x)) = \sqrt[3]{x^3} = x$

(b)



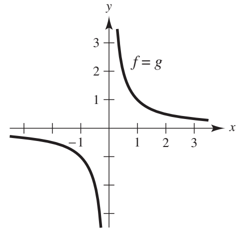
5. (a) $f(g(x)) = \sqrt{x^2 + 4 - 4} = x$;
$g(f(x)) = \left(\sqrt{x - 4}\right)^2 + 4 = x$

(b)



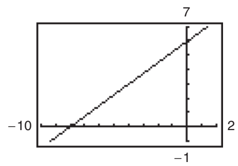
7. (a) $f(g(x)) = \frac{1}{1/x} = x$; $g(f(x)) = \frac{1}{1/x} = x$

(b)



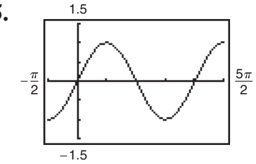
9. c **10.** b **11.** a **12.** d

13.



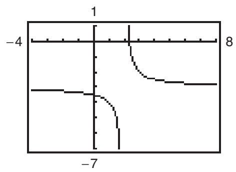
One-to-one, inverse exists.

15.



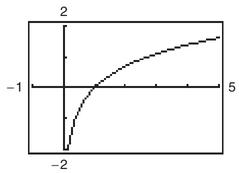
Not one-to-one, inverse does not exist.

17.



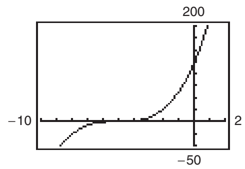
One-to-one, inverse exists.

19.



One-to-one, inverse exists.

21.



One-to-one, inverse exists.

23. Inverse exists. **25.** Inverse does not exist.

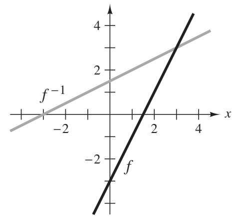
27. Inverse exists. **29.** $f'(x) = 2(x - 4) > 0$ on $(4, \infty)$

31. $f'(x) = -8/x^3 < 0$ on $(0, \infty)$

33. $f'(x) = -\sin x < 0$ on $(0, \pi)$

35. (a) $f^{-1}(x) = (x + 3)/2$

(b)

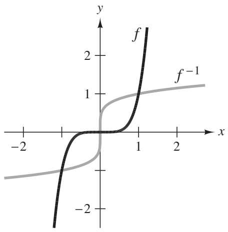


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

37. (a) $f^{-1}(x) = x^{1/5}$

(b)

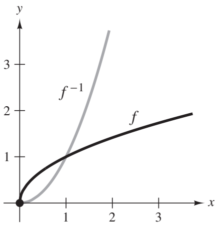


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

39. (a) $f^{-1}(x) = x^2, \quad x \geq 0$

(b)

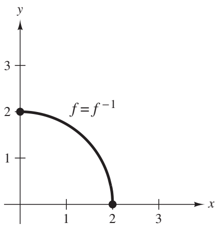


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $x \geq 0$
Range of f and f^{-1}: $y \geq 0$

41. (a) $f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

(b)

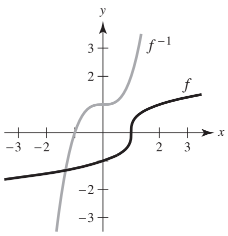


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: $0 \leq x \leq 2$
Range of f and f^{-1}: $0 \leq y \leq 2$

43. (a) $f^{-1}(x) = x^3 + 1$

(b)

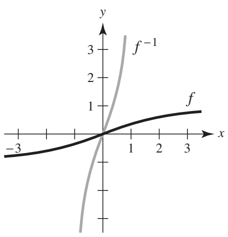


(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

45. (a) $f^{-1}(x) = \sqrt{7}x/\sqrt{1 - x^2}, \quad -1 < x < 1$

(b)



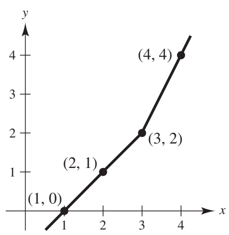
(c) f and f^{-1} are symmetric about $y = x$.

(d) Domain of f: all real numbers
Domain of f^{-1}: $-1 < x < 1$
Range of f: $-1 < y < 1$
Range of f^{-1}: all real numbers

47.

x	0	1	2	4
$f(x)$	1	2	3	4

x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



49. (a) Proof
(b) $y = \frac{20}{7}(80 - x)$
x: total cost
y: number of pounds of the less expensive commodity
(c) $[62.5, 80]$ (d) 20 lb

51. One-to-one
$f^{-1}(x) = x^2 + 2, \quad x \geq 0$

53. One-to-one
$f^{-1}(x) = 2 - x, \quad x \geq 0$

55. Sample answer: $f^{-1}(x) = \sqrt{x} + 3, \quad x \geq 0$

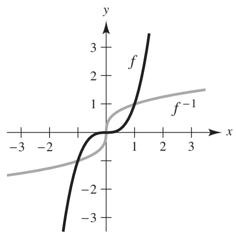
57. Sample answer: $f^{-1}(x) = x - 3, \quad x \geq 0$

59. Inverse exists. Volume is an increasing function, and therefore is one-to-one. The inverse function gives the time t corresponding to the volume V.

61. Inverse does not exist. **63.** $-1/6$ **65.** $1/17$

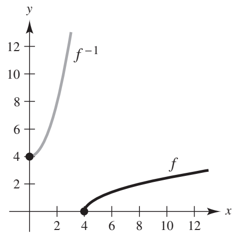
67. $2\sqrt{3}/3$ **69.** -2

71. (a) Domain of f: $(-\infty, \infty)$
Domain of f^{-1}: $(-\infty, \infty)$
(b) Range of f: $(-\infty, \infty)$
Range of f^{-1}: $(-\infty, \infty)$
(c)



(d) $f'\left(\frac{1}{2}\right) = \frac{3}{4}, (f^{-1})'\left(\frac{1}{8}\right) = \frac{4}{3}$

73. (a) Domain of f: $[4, \infty)$
Domain of f^{-1}: $[0. \infty)$
(b) Range of f: $[0, \infty)$
Range of f^{-1}: $[4, \infty)$
(c)



(d) $f'(5) = \frac{1}{2}, (f^{-1})'(1) = 2$

75. 32 **77.** 600 **79.** $(g^{-1} \circ f^{-1})(x) = (x + 1)/2$

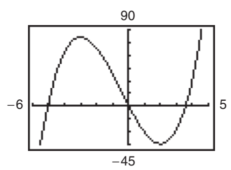
81. $(f \circ g)^{-1}(x) = (x + 1)/2$

83. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
Sample answer: $f(x) = x^3$; $y = x^3$; $x = \sqrt[3]{y}$; $y = \sqrt[3]{x}$; $f^{-1}(x) = \sqrt[3]{x}$

85. Many x-values yield the same y-value. For example, $f(\pi) = 0 = f(0)$. The graph is not continuous at $[(2n - 1)\pi]/2$, where n is an integer.

87. $\frac{1}{4}$ **89.** False. Let $f(x) = x^2$. **91.** True

93. (a)



f does not pass the horizontal line test.

(b) $c = 2$

95–97. Proofs **99.** Proof; concave upward

101. Proof; $\sqrt{5}/5$

103. (a) Proof (b) $f^{-1}(x) = \dfrac{b - dx}{cx - a}$

(c) $a = -d$, or $b = c = 0$, $a = d$

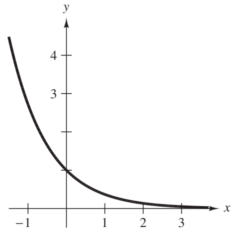
Section 5.4 *(page 352)*

1. $x = 4$ **3.** $x \approx 2.485$ **5.** $x = 0$ **7.** $x \approx 0.511$

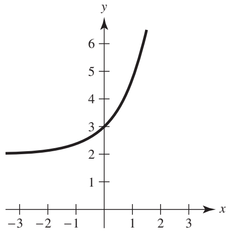
9. $x \approx 8.862$ **11.** $x \approx 7.389$ **13.** $x \approx 10.389$

15. $x \approx 5.389$

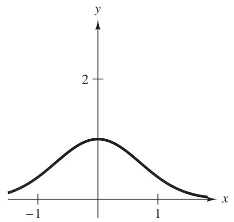
17.



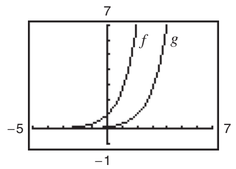
19.



21.

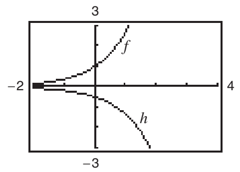


23. (a)



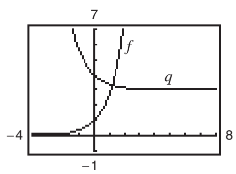
Translation two units to the right

(b)



Reflection in the x-axis and a vertical shrink

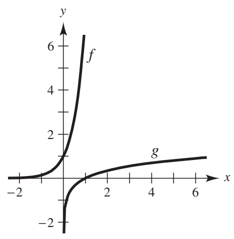
(c)



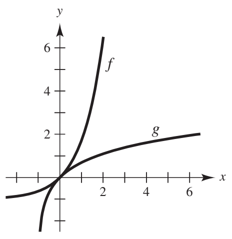
Reflection in the y-axis and a translation three units upward

25. c **26.** d **27.** a **28.** b

29.



31.



33. $2e^{2x}$ **35.** $e^{\sqrt{x}}/(2\sqrt{x})$ **37.** e^{x-4} **39.** $e^x\left(\dfrac{1}{x} + \ln x\right)$

41. $e^x(x^3 + 3x^2)$ **43.** $3(e^{-t} + e^t)^2(e^t - e^{-t})$

45. $2e^{2x}/(1 + e^{2x})$ **47.** $-2(e^x - e^{-x})/(e^x + e^{-x})^2$

49. $-2e^x/(e^x - 1)^2$ **51.** $2e^x \cos x$ **53.** $\cos(x)/x$

55. $y = 3x + 1$ **57.** $y = -x + 2$ **59.** $y = (1/e)x - 1/e$

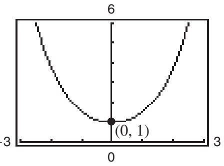
61. $y = ex$ **63.** $\dfrac{10 - e^y}{xe^y + 3}$ **65.** $y = (-e - 1)x + 1$

67. $3(6x + 5)e^{-3x}$

69. $y'' - y = 0$

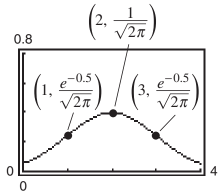
$4e^{-x} - 4e^{-x} = 0$

71. Relative minimum: $(0, 1)$



73. Relative maximum: $\left(2, 1/\sqrt{2\pi}\right)$

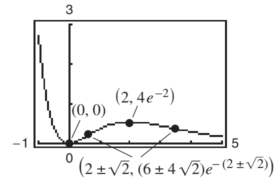
Points of inflection: $\left(1, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right), \left(3, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right)$



75. Relative minimum: $(0, 0)$

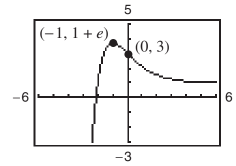
Relative maximum: $(2, 4e^{-2})$

Points of inflection: $\left(2 \pm \sqrt{2}, (6 \pm 4\sqrt{2})e^{-(2 \pm \sqrt{2})}\right)$



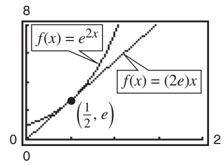
77. Relative maximum: $(-1, 1 + e)$

Point of inflection: $(0, 3)$

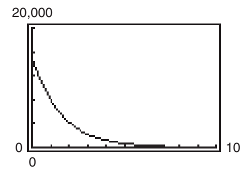


79. $A = \sqrt{2}e^{-1/2}$

81. $\left(\frac{1}{2}, e\right)$



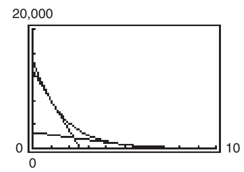
83. (a)



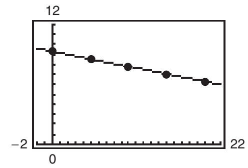
(b) When $t = 1$, $\dfrac{dV}{dt} \approx -5028.84$.

When $t = 5$, $\dfrac{dV}{dt} \approx -406.89$.

(c)



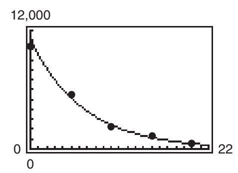
85. (a)



$\ln P = -0.1499h + 9.3018$

(b) $P = 10{,}957.7e^{-0.1499h}$

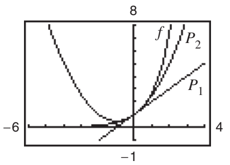
(c)



(d) $h = 5$: -776

$h = 18$: -111

87. $P_1 = 1 + x$; $P_2 = 1 + x + \frac{1}{2}x^2$



The values of f, P_1, and P_2 and their first derivatives agree at $x = 0$.

89. $12! = 479{,}001{,}600$
Stirling's Formula: $12! \approx 475{,}687{,}487$

91. $e^{5x} + C$ **93.** $\frac{1}{2}e^{2x-1} + C$ **95.** $\frac{1}{3}e^{x^3} + C$

97. $2e^{\sqrt{x}} + C$

99. $x - \ln(e^x + 1) + C_1$ or $-\ln(1 + e^{-x}) + C_2$

101. $-\frac{2}{3}(1 - e^x)^{3/2} + C$ **103.** $\ln|e^x - e^{-x}| + C$

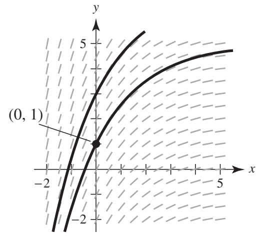
105. $-\frac{5}{2}e^{-2x} + e^{-x} + C$ **107.** $\ln|\cos e^{-x}| + C$

109. $(e^2 - 1)/(2e^2)$ **111.** $(e - 1)/(2e)$

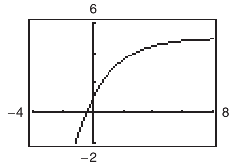
113. $(e/3)(e^2 - 1)$ **115.** $\ln\left(\dfrac{1 + e^6}{2}\right)$

117. $(1/\pi)[e^{\sin(\pi^2/2)} - 1]$

119. (a)

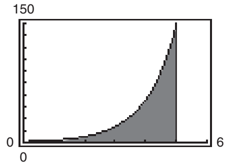


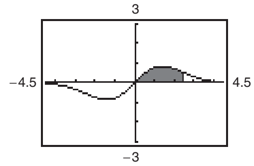
(b) $y = -4e^{-x/2} + 5$



121. $[1/(2a)]e^{ax^2} + C$ **123.** $f(x) = \frac{1}{2}(e^x + e^{-x})$

125. $e^5 - 1 \approx 147.413$ **127.** $2(1 - e^{-3/2}) \approx 1.554$





129. Midpoint Rule: 92.190; Trapezoidal Rule: 93.837;
Simpson's Rule: 92.7385

131. The probability that a given battery will last between 48 months and 60 months is approximately 47.72%.

133. $a = \ln 3$

135. $f(x) = e^x$
The domain of $f(x)$ is $(-\infty, \infty)$, and the range of $f(x)$ is $(0, \infty)$. $f(x)$ is continuous, increasing, one-to-one, and concave upward on its entire domain.
$\lim_{x\to-\infty} e^x = 0$ and $\lim_{x\to\infty} e^x = \infty$

137. (a) Log Rule (b) Substitution

139. $\int_0^x e^t\,dt \ge \int_0^x 1\,dt$; $e^x - 1 \ge x$; $e^x \ge x + 1$ for $x \ge 0$

141. (a) $t = \dfrac{1}{2k}\ln\dfrac{B}{A}$
(b) $x''(t) = k^2(Ae^{kt} + Be^{-kt})$, k^2 is the constant of proportionality.

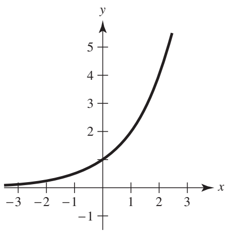
143. Proof

Section 5.5 *(page 362)*

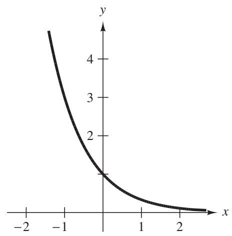
1. -3 **3.** 0 **5.** (a) $\log_2 8 = 3$ (b) $\log_3(1/3) = -1$

7. (a) $10^{-2} = 0.01$ (b) $\left(\frac{1}{2}\right)^{-3} = 8$

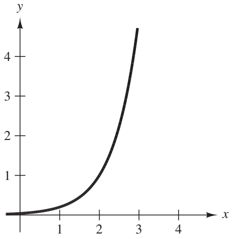
9.



11.



13.



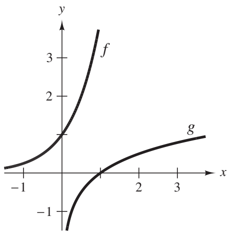
15. d **16.** c **17.** b **18.** a

19. (a) $x = 3$ (b) $x = -1$ **21.** (a) $x = \frac{1}{3}$ (b) $x = \frac{1}{16}$

23. (a) $x = -1, 2$ (b) $x = \frac{1}{3}$ **25.** 1.965 **27.** -6.288

29. 12.253 **31.** 33.000 **33.** ± 11.845

35.



37. $(\ln 4)4^x$ **39.** $(-4\ln 5)5^{-4x}$ **41.** $9^x(x\ln 9 + 1)$

43. $t2^t(t\ln 2 + 2)$ **45.** $-2^{-\theta}[(\ln 2)\cos\pi\theta + \pi\sin\pi\theta]$

47. $5/[(\ln 4)(5x + 1)]$ **49.** $2/[(\ln 5)(t - 4)]$

51. $x/[(\ln 5)(x^2 - 1)]$ **53.** $(x - 2)/[(\ln 2)x(x - 1)]$

55. $(3x - 2)/[(2x\ln 3)(x - 1)]$ **57.** $5(1 - \ln t)/(t^2\ln 2)$

59. $y = -2x\ln 2 - 2\ln 2 + 2$

61. $y = [1/(27\ln 3)]x + 3 - 1/\ln 3$ **63.** $2(1 - \ln x)x^{(2/x)-2}$

65. $(x - 2)^{x+1}[(x + 1)/(x - 2) + \ln(x - 2)]$

67. $y = x$ **69.** $y = \dfrac{\cos e}{e}x - \cos e + 1$

71. $3^x/\ln 3 + C$ **73.** $\dfrac{1}{3}x^3 - \dfrac{2^{-x}}{\ln 2} + C$

75. $[-1/(2\ln 5)](5^{-x^2}) + C$ **77.** $\ln(3^{2x} + 1)/(2\ln 3) + C$

79. $7/(2\ln 2)$ **81.** $4/\ln 5 - 2/\ln 3$ **83.** $26/\ln 3$

85. (a) $x > 0$ (b) 10^x (c) $3 \le f(x) \le 4$
(d) $0 < x < 1$ (e) 10 (f) 100^n

87. (a) \$40.64 (b) $C'(1) \approx 0.051P$, $C'(8) \approx 0.072P$
(c) ln 1.05

89.

n	1	2	4	12
A	\$1410.60	\$1414.78	\$1416.91	\$1418.34

n	365	Continuous
A	\$1419.04	\$1419.07

91.

n	1	2	4	12
A	\$4321.94	\$4399.79	\$4440.21	\$4467.74

n	365	Continuous
A	\$4481.23	\$4481.69

93.

t	1	10	20	30
P	\$95,122.94	\$60,653.07	\$36,787.94	\$22,313.02

t	40	50
P	\$13,533.53	\$8208.50

95.

t	1	10	20	30
P	\$95,132.82	\$60,716.10	\$36,864.45	\$22,382.66

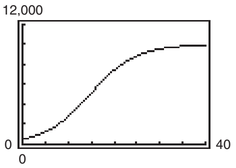
t	40	50
P	\$13,589.88	\$8251.24

97. c

99. (a) 6.7 million ft^3/acre

(b) $t = 20$: $\dfrac{dV}{dt} = 0.073$; $t = 60$: $\dfrac{dV}{dt} = 0.040$

101. (a)



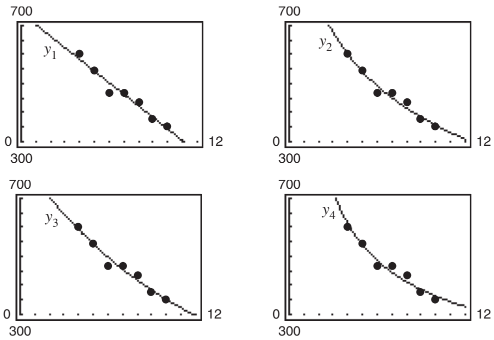
(b) 10,000 fish

(c) 1 month: About 114 fish/mo
10 months: About 403 fish/mo

(d) About 15 mo

103. (a) $y_1 = -40x + 743$, $y_2 = 968 - 265.5 \ln x$,
$y_3 = 836.817(0.9169)^x$, $y_4 = 1344.8884x^{-0.5689}$

(b)



(c) The number of pancreas transplants is decreasing by about 40 transplants each year.

(d) $y_1'(8) = -40.04$, $y_2'(8) = -33.18$, $y_3'(8) = -36.27$, $y_4'(8) = -29.30$; y_1 is decreasing at the greatest rate.

105. $y = 1200(0.6^t)$ **107.** e **109.** e^2

111. False. e is an irrational number. **113.** True **115.** True

117. (a) $(2^3)^2 = 2^6 = 64$
$2^{(3^2)} = 2^9 = 512$

(b) No. $f(x) = (x^x)^x = x^{(x^2)}$ and $g(x) = x^{(x^x)}$

(c) $f'(x) = x^{x^2}(x + 2x \ln x)$
$g'(x) = x^{x^x + x - 1}[x(\ln x)^2 + x \ln x + 1]$

119. Proof

121. (a) $\dfrac{dy}{dx} = \dfrac{y^2 - yx \ln y}{x^2 - xy \ln x}$

(b) (i) 1 when $c \neq 0$, $c \neq e$ (ii) -3.1774
(iii) -0.3147

(c) (e, e)

123. Putnam Problem B3, 1951

Section 5.6 *(page 372)*

1. $\left(-\sqrt{2}/2, 3\pi/4\right), (1/2, \pi/3), \left(\sqrt{3}/2, \pi/6\right)$ **3.** $\pi/6$

5. $\pi/3$ **7.** $\pi/6$ **9.** $-\pi/4$ **11.** 2.50

13. $\arccos(1/1.269) \approx 0.66$ **15.** x **17.** $\sqrt{1 - x^2}/x$

19. $1/x$ **21.** (a) $3/5$ (b) $5/3$

23. (a) $-\sqrt{3}$ (b) $-\frac{13}{5}$ **25.** $\sqrt{1 - 4x^2}$

27. $\sqrt{x^2 - 1}/|x|$ **29.** $\sqrt{x^2 - 9}/3$ **31.** $\sqrt{x^2 + 2}/x$

33. $x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$ **35.** $x = \frac{1}{3}$

37. (a) and (b) Proofs **39.** $2/\sqrt{2x - x^2}$

41. $-3/\sqrt{4 - x^2}$ **43.** $e^x/(1 + e^{2x})$

45. $\left(3x - \sqrt{1 - 9x^2} \arcsin 3x\right)/\left(x^2\sqrt{1 - 9x^2}\right)$

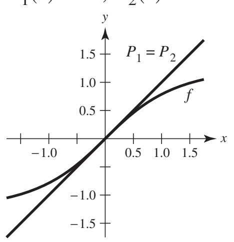
47. $-t/\sqrt{1 - t^2}$ **49.** $2 \arccos x$ **51.** $1/(1 - x^4)$

53. $\arcsin x$ **55.** $x^2/\sqrt{16 - x^2}$ **57.** $2/(1 + x^2)^2$

59. $y = \frac{1}{3}\left(4\sqrt{3}x - 2\sqrt{3} + \pi\right)$ **61.** $y = \frac{1}{4}x + (\pi - 2)/4$

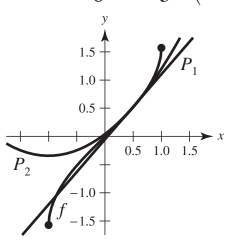
63. $y = (2\pi - 4)x + 4$

65. $P_1(x) = x$; $P_2(x) = x$



67. $P_1(x) = \dfrac{\pi}{6} + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{1}{2}\right)$

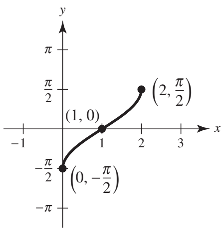
$P_2(x) = \dfrac{\pi}{6} + \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{1}{2}\right) + \dfrac{2\sqrt{3}}{9}\left(x - \dfrac{1}{2}\right)^2$



69. Relative maximum: $(1.272, -0.606)$
Relative minimum: $(-1.272, 3.747)$

71. Relative maximum: $(2, 2.214)$

73.

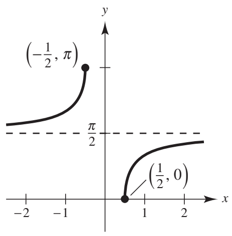


Maximum: $\left(2, \frac{\pi}{2}\right)$

Minimum: $\left(0, -\frac{\pi}{2}\right)$

Point of inflection: $(1, 0)$

75.



Maximum: $\left(-\frac{1}{2}, \pi\right)$

Minimum: $\left(\frac{1}{2}, 0\right)$

Asymptote: $y = \frac{\pi}{2}$

77. $y = -2\pi x/(\pi + 8) + 1 - \pi^2/(2\pi + 16)$

79. $y = -x + \sqrt{2}$

81. If the domains were not restricted, then the trigonometric functions would have no inverses, because they would not be one-to-one.

83. (a) $\arcsin(\arcsin 0.5) \approx 0.551$
$\arcsin(\arcsin 1)$ does not exist.
(b) $\sin(-1) \le x \le \sin(1)$

85. False. The range of arccos is $[0, \pi]$.

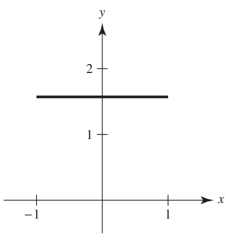
87. True **89.** True

91. (a) $\theta = \operatorname{arccot}(x/5)$
(b) $x = 10$: 16 rad/h; $x = 3$: 58.824 rad/h

93. (a) $h(t) = -16t^2 + 256$; $t = 4$ sec
(b) $t = 1$: -0.0520 rad/sec; $t = 2$: -0.1116 rad/sec

95. $50\sqrt{2} \approx 70.71$ ft **97.** (a) and (b) Proofs

99. (a)



(b) The graph is a horizontal line at $\frac{\pi}{2}$.
(c) Proof

101. $c = 2$ **103.** Proof

Section 5.7 *(page 380)*

1. $\arcsin \frac{x}{3} + C$ **3.** $\operatorname{arcsec}|2x| + C$

5. $\arcsin(x + 1) + C$ **7.** $\frac{1}{2}\arcsin t^2 + C$

9. $\frac{1}{10}\arctan \frac{t^2}{5} + C$ **11.** $\frac{1}{4}\arctan(e^{2x}/2) + C$

13. $\arcsin\left(\frac{\tan x}{5}\right) + C$ **15.** $2\arcsin\sqrt{x} + C$

17. $\frac{1}{2}\ln(x^2 + 1) - 3\arctan x + C$

19. $8\arcsin[(x - 3)/3] - \sqrt{6x - x^2} + C$ **21.** $\pi/6$

23. $\pi/6$ **25.** $\frac{1}{5}\arctan\frac{3}{5} \approx 0.108$

27. $\arctan 5 - \pi/4 \approx 0.588$ **29.** $\pi/4$ **31.** $\frac{1}{32}\pi^2 \approx 0.308$

33. $\pi/2$ **35.** $\ln|x^2 + 6x + 13| - 3\arctan[(x + 3)/2] + C$

37. $\arcsin[(x + 2)/2] + C$ **39.** $4 - 2\sqrt{3} + \frac{1}{6}\pi \approx 1.059$

41. $\frac{1}{2}\arctan(x^2 + 1) + C$

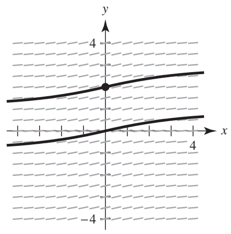
43. $2\sqrt{e^t - 3} - 2\sqrt{3}\arctan(\sqrt{e^t - 3}/\sqrt{3}) + C$ **45.** $\pi/6$

47. a and b **49.** a, b, and c

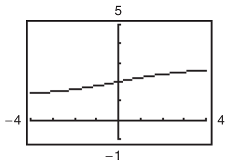
51. No. This integral does not correspond to any of the basic integration rules.

53. $y = \arcsin(x/2) + \pi$

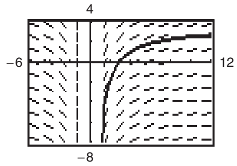
55. (a)



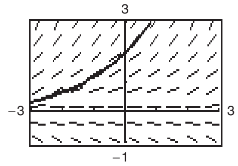
(b) $y = \frac{2}{3}\arctan\frac{x}{3} + 2$



57.

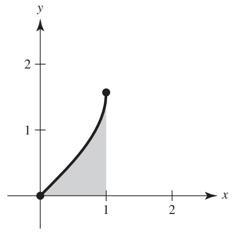


59.



61. $\pi/3$ **63.** $\pi/8$ **65.** $3\pi/2$

67. (a)



(b) 0.5708
(c) $(\pi - 2)/2$

69. (a) $F(x)$ represents the average value of $f(x)$ over the interval $[x, x + 2]$. Maximum at $x = -1$
(b) Maximum at $x = -1$

71. False. $\displaystyle\int \frac{dx}{3x\sqrt{9x^2 - 16}} = \frac{1}{12}\operatorname{arcsec}\frac{|3x|}{4} + C$

73. True **75–77.** Proofs

79. (a) $\displaystyle\int_0^1 \frac{1}{1 + x^2}\,dx$ (b) About 0.7847
(c) Because $\displaystyle\int_0^1 \frac{1}{1 + x^2}\,dx = \frac{\pi}{4}$, you can use the Trapezoidal Rule to approximate $\frac{\pi}{4}$. Multiplying the result by 4 gives an estimation of π.

Section 5.8 *(page 390)*

1. (a) 10.018 (b) -0.964 **3.** (a) $\frac{4}{3}$ (b) $\frac{13}{12}$

5. (a) 1.317 (b) 0.962 **7–13.** Proofs

15. $\cosh x = \sqrt{13}/2$; $\tanh x = 3\sqrt{13}/13$; $\operatorname{csch} x = 2/3$; $\operatorname{sech} x = 2\sqrt{13}/13$; $\coth x = \sqrt{13}/3$

17. ∞ **19.** 0 **21.** 1 **23.** $3\cosh 3x$

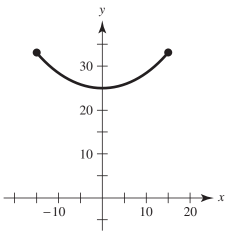
25. $-10x[\operatorname{sech}(5x^2)\tanh(5x^2)]$ **27.** $\coth x$ **29.** $\sinh^2 x$

31. $\operatorname{sech} t$ **33.** $y = -2x + 2$ **35.** $y = 1 - 2x$

37. Relative maxima: $(\pm\pi, \cosh \pi)$; Relative minimum: $(0, -1)$

39. Relative maximum: $(1.20, 0.66)$;
Relative minimum: $(-1.20, -0.66)$

41. (a)

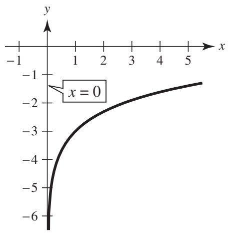


(b) 33.146 units; 25 units
(c) $m = \sinh(1) \approx 1.175$

43. $\frac{1}{2}\sinh 2x + C$ **45.** $-\frac{1}{2}\cosh(1 - 2x) + C$
47. $\frac{1}{3}\cosh^3(x - 1) + C$ **49.** $\ln|\sinh x| + C$
51. $-\coth(x^2/2) + C$ **53.** $\operatorname{csch}(1/x) + C$ **55.** $\ln(5/4)$
57. $\frac{1}{5}\ln 3$ **59.** $\pi/4$ **61.** Answers will vary.
63. $\cosh x$, $\operatorname{sech} x$ **65.** $3/\sqrt{9x^2 - 1}$ **67.** $\dfrac{1}{2\sqrt{x}(1 - x)}$
69. $|\sec x|$ **71.** $\dfrac{-2\operatorname{csch}^{-1} x}{|x|\sqrt{1 + x^2}}$ **73.** $2\sinh^{-1}(2x)$
75. $\dfrac{\sqrt{3}}{18}\ln\left|\dfrac{1 + \sqrt{3}x}{1 - \sqrt{3}x}\right| + C$ **77.** $\ln\left(\sqrt{e^{2x} + 1} - 1\right) - x + C$
79. $2\sinh^{-1}\sqrt{x} + C = 2\ln\left(\sqrt{x} + \sqrt{1 + x}\right) + C$
81. $\dfrac{1}{4}\ln\left|\dfrac{x - 4}{x}\right| + C$ **83.** $\ln\left(\dfrac{3 + \sqrt{5}}{2}\right)$ **85.** $\dfrac{\ln 7}{12}$
87. $\dfrac{1}{4}\arcsin\left(\dfrac{4x - 1}{9}\right) + C$
89. $-\dfrac{x^2}{2} - 4x - \dfrac{10}{3}\ln\left|\dfrac{x - 5}{x + 1}\right| + C$
91. $8\arctan(e^2) - 2\pi \approx 5.207$ **93.** $\frac{5}{2}\ln\left(\sqrt{17} + 4\right) \approx 5.237$
95. $\frac{52}{31}$ kg **97.** (a) $-\sqrt{a^2 - x^2}/x$ (b) Proof
99–107. Proofs **109.** Putnam Problem 8, 1939

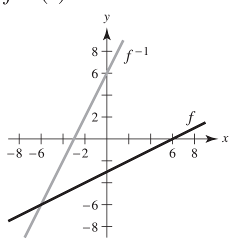
Review Exercises for Chapter 5 *(page 393)*

1.



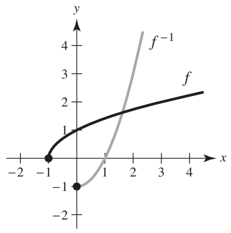
Domain: $x > 0$

3. $\frac{1}{5}[\ln(2x + 1) + \ln(2x - 1) - \ln(4x^2 + 1)]$
5. $\ln\left(3\sqrt[3]{4 - x^2}/x\right)$ **7.** $1/(2x)$ **9.** $(1 + 2\ln x)/\left(2\sqrt{\ln x}\right)$
11. $-\dfrac{8x}{x^4 - 16}$ **13.** $y = -x + 1$ **15.** $\dfrac{1}{7}\ln|7x - 2| + C$
17. $-\ln|1 + \cos x| + C$ **19.** $3 + \ln 2$ **21.** $\ln\left(2 + \sqrt{3}\right)$
23. (a) $f^{-1}(x) = 2x + 6$
(b)



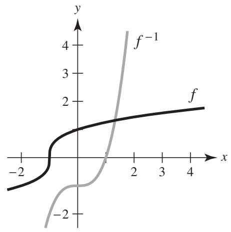
(c) Proof
(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

25. (a) $f^{-1}(x) = x^2 - 1, \quad x \ge 0$
(b)



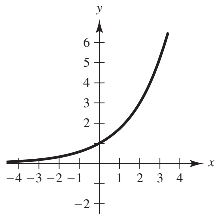
(c) Proof
(d) Domain of f: $x \ge -1$; Domain of f^{-1}: $x \ge 0$
Range of f: $y \ge 0$; Range of f^{-1}: $y \ge -1$

27. (a) $f^{-1}(x) = x^3 - 1$
(b)

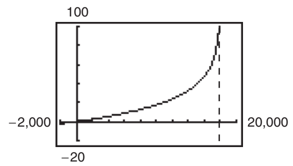


(c) Proof
(d) Domain of f and f^{-1}: all real numbers
Range of f and f^{-1}: all real numbers

29. $1/\left[3\left(\sqrt[3]{-3}\right)^2\right] \approx 0.160$ **31.** $3/4$ **33.** $x \approx 1.134$
35. $e^4 - 1 \approx 53.598$ **37.** $te^t(t + 2)$
39. $(e^{2x} - e^{-2x})/\sqrt{e^{2x} + e^{-2x}}$ **41.** $x(2 - x)/e^x$
43. $y = 6x + 1$ **45.** $-y/[x(2y + \ln x)]$ **47.** $-\frac{1}{2}e^{1 - x^2} + C$
49. $(e^{4x} - 3e^{2x} - 3)/(3e^x) + C$ **51.** $(1 - e^{-3})/6 \approx 0.158$
53. $\ln(e^2 + e + 1) \approx 2.408$ **55.** About 1.729
57.



59. $3^{x-1}\ln 3$ **61.** $x^{2x+1}(2\ln x + 2 + 1/x)$
63. $-1/[\ln 3(2 - 2x)]$ **65.** $5^{(x+1)^2}/(2\ln 5) + C$
67. (a) Domain: $0 \le h < 18{,}000$
(b)



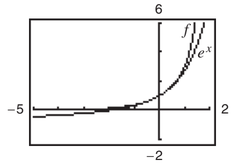
(c) $t = 0$
Vertical asymptote: $h = 18{,}000$

69. (a) $1/2$ (b) $\sqrt{3}/2$ **71.** $(1 - x^2)^{-3/2}$
73. $\dfrac{x}{|x|\sqrt{x^2 - 1}} + \operatorname{arcsec} x$ **75.** $(\arcsin x)^2$
77. $\frac{1}{2}\arctan(e^{2x}) + C$ **79.** $\frac{1}{2}\arcsin x^2 + C$
81. $\frac{1}{4}[\arctan(x/2)]^2 + C$ **83.** $\frac{2}{3}\pi + \sqrt{3} - 2 \approx 1.826$
85. $y' = -4\operatorname{sech}(4x - 1)\tanh(4x - 1)$
87. $y' = -16x\operatorname{csch}^2(8x^2)$ **89.** $y' = \dfrac{4}{\sqrt{16x^2 + 1}}$
91. $\frac{1}{3}\tanh x^3 + C$ **93.** $\ln|\tanh x| + C$
95. $\dfrac{1}{12}\ln\left|\dfrac{3 + 2x}{3 - 2x}\right| + C$

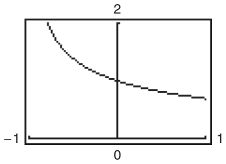
P.S. Problem Solving *(page 395)*

1. $a = 1, b = \frac{1}{2}, c = -\frac{1}{2}$

$f(x) = \dfrac{1 + x/2}{1 - x/2}$

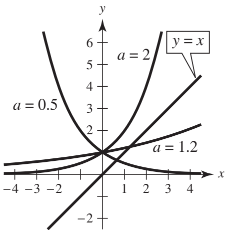


3. (a)



(b) 1 (c) Proof

5.



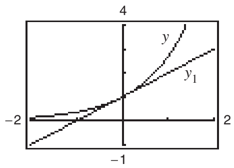
$y = 0.5^x$ and $y = 1.2^x$ intersect the line $y = x$; $0 < a < e^{1/e}$

7. $e - 1$

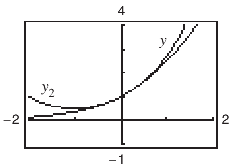
9. (a) Area of region $A = (\sqrt{3} - \sqrt{2})/2 \approx 0.1589$
Area of region $B = \pi/12 \approx 0.2618$
(b) $\frac{1}{24}[3\pi\sqrt{2} - 12(\sqrt{3} - \sqrt{2}) - 2\pi] \approx 0.1346$
(c) 1.2958 (d) 0.6818

11. Proof **13.** $2 \ln \frac{3}{2} \approx 0.8109$

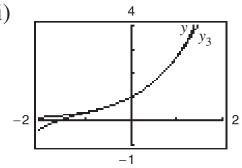
15. (a) (i)



(ii)

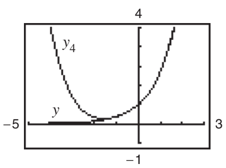


(iii)



(b) Pattern: $y_n = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \cdots + \dfrac{x^n}{n!} + \cdots$

$y_4 = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!}$



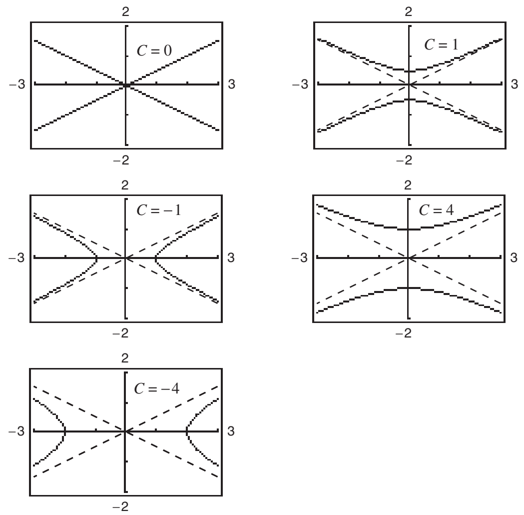
(c) The pattern implies that $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$.

Chapter 6

Section 6.1 *(page 403)*

1–11. Proofs **13.** Not a solution **15.** Solution
17. Solution **19.** Solution **21.** Not a solution
23. Solution **25.** Not a solution **27.** Not a solution
29. $y = 3e^{-x/2}$ **31.** $4y^2 = x^3$

33.



35. $y = 3e^{-2x}$ **37.** $y = 2 \sin 3x - \frac{1}{3} \cos 3x$
39. $y = -2x + \frac{1}{2}x^3$ **41.** $2x^3 + C$
43. $y = \frac{1}{2} \ln(1 + x^2) + C$ **45.** $y = x - \ln x^2 + C$
47. $y = -\frac{1}{2} \cos 2x + C$
49. $y = \frac{2}{5}(x - 6)^{5/2} + 4(x - 6)^{3/2} + C$ **51.** $y = \frac{1}{2}e^{x^2} + C$

53.

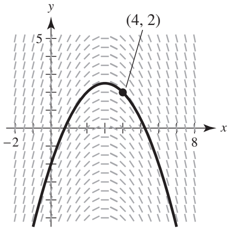
x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	-4	Undef.	0	1	$\frac{4}{3}$	2

55.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

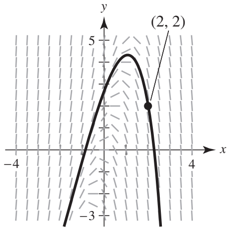
57. b **58.** c **59.** d **60.** a

61. (a) and (b)



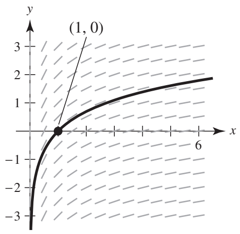
(c) As $x \to \infty, y \to -\infty$; as $x \to -\infty, y \to -\infty$

63. (a) and (b)



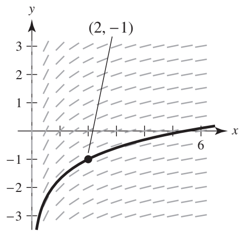
(c) As $x \to \infty, y \to -\infty$; as $x \to -\infty, y \to -\infty$

65. (a)



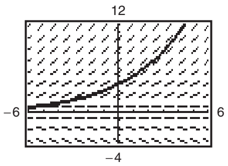
As $x \to \infty$, $y \to \infty$

(b)

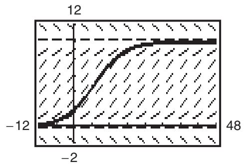


As $x \to \infty$, $y \to \infty$

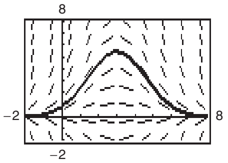
67. (a) and (b)



69. (a) and (b)



71. (a) and (b)



73.

n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	2	2.2	2.43	2.693	2.992	3.332	3.715

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	4.146	4.631	5.174	5.781

75.

n	0	1	2	3	4	5	6
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3
y_n	3	2.7	2.438	2.209	2.010	1.839	1.693

n	7	8	9	10
x_n	0.35	0.4	0.45	0.5
y_n	1.569	1.464	1.378	1.308

77.

n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	1	1.1	1.212	1.339	1.488	1.670	1.900

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	2.213	2.684	3.540	5.958

79.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	3.0000	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ $(h = 0.2)$	3.0000	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ $(h = 0.1)$	3.0000	3.6300	4.3923	5.3147	6.4308	7.7812

81.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	0.0000	0.2200	0.4801	0.7807	1.1231	1.5097
$y(x)$ $(h = 0.2)$	0.0000	0.2000	0.4360	0.7074	1.0140	1.3561
$y(x)$ $(h = 0.1)$	0.0000	0.2095	0.4568	0.7418	1.0649	1.4273

83. (a) $y(1) = 112.7141°$; $y(2) = 96.3770°$; $y(3) = 86.5954°$
(b) $y(1) = 113.2441°$; $y(2) = 97.0158°$; $y(3) = 87.1729°$
(c) Euler's Method: $y(1) = 112.9828°$; $y(2) = 96.6998°$; $y(3) = 86.8863°$
Exact solution: $y(1) = 113.2441°$; $y(2) = 97.0158°$; $y(3) = 87.1729°$
The approximations are better using $h = 0.05$.

85. The general solution is a family of curves that satisfies the differential equation. A particular solution is one member of the family that satisfies given conditions.

87. Begin with a point (x_0, y_0) that satisfies the initial condition $y(x_0) = y_0$. Then, using a small step size h, calculate the point $(x_1, y_1) = (x_0 + h, y_0 + hF(x_0, y_0))$. Continue generating the sequence of points $(x_n + h, y_n + hF(x_n, y_n))$ or (x_{n+1}, y_{n+1}).

89. False. $y = x^3$ is a solution of $xy' - 3y = 0$, but $y = x^3 + 1$ is not a solution.

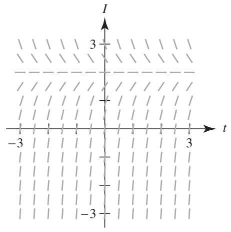
91. True

93. (a)

x	0	0.2	0.4	0.6	0.8	1
y	4	2.6813	1.7973	1.2048	0.8076	0.5413
y_1	4	2.56	1.6384	1.0486	0.6711	0.4295
y_2	4	2.4	1.44	0.864	0.5184	0.3110
e_1	0	0.1213	0.1589	0.1562	0.1365	0.1118
e_2	0	0.2813	0.3573	0.3408	0.2892	0.2303
r		0.4312	0.4447	0.4583	0.4720	0.4855

(b) If h is halved, then the error is approximately halved because r is approximately 0.5.
(c) The error will again be halved.

95. (a)



(b) $\lim_{t\to\infty} I(t) = 2$

97. $\omega = \pm 4$ **99.** Putnam Problem 3, Morning Session, 1954

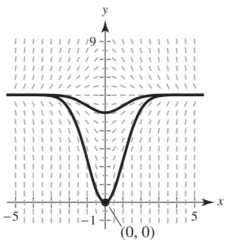
Section 6.2 *(page 412)*

1. $y = \frac{1}{2}x^2 + 3x + C$ **3.** $y = Ce^x - 3$

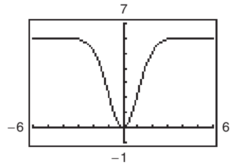
5. $y^2 - 5x^2 = C$ **7.** $y = Ce^{(2x^{3/2})/3}$ **9.** $y = C(1 + x^2)$

11. $dQ/dt = k/t^2$
$Q = -k/t + C$

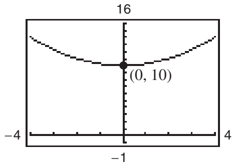
13. (a)



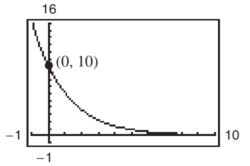
(b) $y = 6 - 6e^{-x^2/2}$



15. $y = \frac{1}{4}t^2 + 10$



17. $y = 10e^{-t/2}$



19. $\frac{8192}{4}$ **21.** $y = (1/2)e^{[(\ln 10)/5]t} \approx (1/2)e^{0.4605t}$

23. $y = 5(5/2)^{1/4}e^{[\ln(2/5)/4]t} \approx 6.2872e^{-0.2291t}$

25. C is the initial value of y, and k is the proportionality constant.

27. Quadrants I and III; dy/dx is positive when both x and y are positive (Quadrant I) or when both x and y are negative (Quadrant III).

29. Amount after 1000 yr: 12.96 g;
Amount after 10,000 yr: 0.26 g

31. Initial quantity: 7.63 g;
Amount after 1000 yr: 4.95 g

33. Amount after 1000 yr: 4.43 g;
Amount after 10,000 yr: 1.49 g

35. Initial quantity: 2.16 g;
Amount after 10,000 yr: 1.62 g

37. 95.76%

39. Time to double: 11.55 yr; Amount after 10 yr: \$7288.48

41. Annual rate: 8.94%; Amount after 10 yr: \$1833.67

43. Annual rate: 9.50%; Time to double: 7.30 yr

45. \$224,174.18 **47.** \$61,377.75

49. (a) 10.24 yr (b) 9.93 yr (c) 9.90 yr (d) 9.90 yr

51. (a) $P = 2.21e^{-0.006t}$ (b) 2.08 million
(c) Because $k < 0$, the population is decreasing.

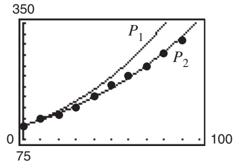
53. (a) $P = 33.38e^{0.036t}$ (b) 47.84 million
(c) Because $k > 0$, the population is increasing.

55. (a) $N = 100.1596(1.2455)^t$ (b) 6.3 h

57. (a) $N \approx 30(1 - e^{-0.0502t})$ (b) 36 days

59. (a) Because the population increases by a constant each month, the rate of change from month to month will always be the same. So, the slope is constant, and the model is linear.
(b) Although the percentage increase is constant each month, the rate of growth is not constant. The rate of change of y is $dy/dt = ry$, which is an exponential model.

61. (a) $P_1 = 106e^{0.01487t} \approx 106(1.01499)^t$
(b) $P_2 = 107.2727(1.01215)^t$
(c)



(d) 2029

63. (a) 20 dB (b) 70 dB (c) 95 dB (d) 120 dB

65. 379.2°F

67. False. The rate of growth dy/dx is proportional to y.

69. False. The prices are rising at a rate of 6.2% per year.

Section 6.3 *(page 421)*

1. $y^2 - x^2 = C$ **3.** $15y^2 + 2x^3 = C$ **5.** $r = Ce^{0.75s}$

7. $y = C(x + 2)^3$ **9.** $y^2 = C - 8\cos x$

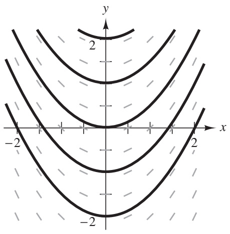
11. $y = -\frac{1}{4}\sqrt{1 - 4x^2} + C$ **13.** $y = Ce^{(\ln x)^2/2}$

15. $y^2 = 4e^x + 5$ **17.** $y = e^{-(x^2+2x)/2}$

19. $y^2 = 4x^2 + 3$ **21.** $u = e^{(1-\cos v^2)/2}$ **23.** $P = P_0e^{kt}$

25. $4y^2 - x^2 = 16$ **27.** $y = \frac{1}{3}\sqrt{x}$ **29.** $f(x) = Ce^{-x/2}$

31.



$y = \frac{1}{2}x^2 + C$

33. (a) $dy/dx = k(y - 4)$ (b) a (c) Proof

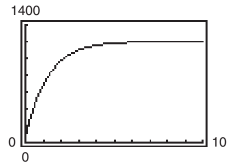
34. (a) $dy/dx = k(x - 4)$ (b) b (c) Proof

35. (a) $dy/dx = ky(y - 4)$ (b) c (c) Proof

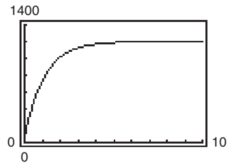
36. (a) $dy/dx = ky^2$ (b) d (c) Proof

37. 97.9% of the original amount

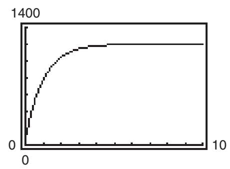
39. (a) $w = 1200 - 1140e^{-kt}$
(b) $w = 1200 - 1140e^{-0.8t}$



$w = 1200 - 1140e^{-0.9t}$

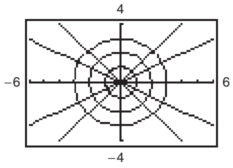


$w = 1200 - 1140e^{-t}$

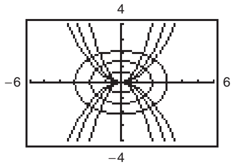


(c) 1.31 yr; 1.16 yr; 1.05 yr (d) 1200 lb

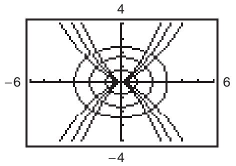
41. Circles: $x^2 + y^2 = C$
Lines: $y = Kx$
Graphs will vary.



43. Parabolas: $x^2 = Cy$
Ellipses: $x^2 + 2y^2 = K$
Graphs will vary.



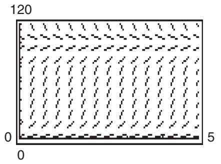
45. Curves: $y^2 = Cx^3$
Ellipses: $2x^2 + 3y^2 = K$
Graphs will vary.



47. d **48.** a **49.** b **50.** c

51. (a) 0.75 (b) 2100 (c) 70 (d) 4.49 yr
(e) $dP/dt = 0.75P(1 - P/2100)$

53. (a) 3 (b) 100
(c)



(d) 50

55. $y = 36/(1 + 8e^{-t})$ **57.** $y = 120/(1 + 14e^{-0.8t})$

59. (a) $P = \dfrac{200}{1 + 7e^{-0.2640t}}$ (b) 70 panthers (c) 7.37 yr
(d) $dP/dt = 0.2640P(1 - P/200)$; 65.6 (e) 100 yr

61. Answers will vary. **63.** Proof

65. (a) $v = 20(1 - e^{-1.386t})$
(b) $s \approx 20t + 14.43(e^{-1.386t} - 1)$

67. Homogeneous of degree 3 **69.** Homogeneous of degree 3

71. Not homogeneous **73.** Homogeneous of degree 0.

75. $|x| = C(x - y)^2$ **77.** $|y^2 + 2xy - x^2| = C$

79. $y = Ce^{-x^2/(2y^2)}$

81. False. $y' = x/y$ is separable, but $y = 0$ is not a solution.

83. True

Section 6.4 *(page 428)*

1. Linear; can be written in the form $dy/dx + P(x)y = Q(x)$

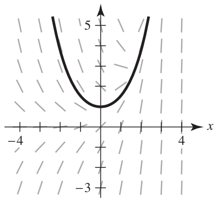
3. Not linear; cannot be written in the form $dy/dx + P(x)y = Q(x)$

5. $y = 2x^2 + x + C/x$ **7.** $y = -16 + Ce^x$

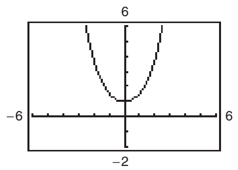
9. $y = -1 + Ce^{\sin x}$ **11.** $y = (x^3 - 3x + C)/[3(x - 1)]$

13. $y = e^{x^3}(x + C)$

15. (a) Answers will vary.



(b) $y = \frac{1}{2}(e^x + e^{-x})$
(c)



17. $y = 1 + 4/e^{\tan x}$ **19.** $y = \sin x + (x + 1)\cos x$

21. $xy = 4$ **23.** $y = -2 + x\ln|x| + 12x$

25. $P = -N/k + (N/k + P_0)e^{kt}$

27. (a) \$4,212,796.94 (b) \$31,424,909.75

29. (a) $\dfrac{dN}{dt} = k(75 - N)$ (b) $N = 75 + Ce^{-kt}$
(c) $N = 75 - 55.9296e^{-0.0168t}$

31. $v(t) = -159.47(1 - e^{-0.2007t})$; -159.47 ft/sec

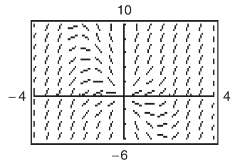
33. $I = \dfrac{E_0}{R} + Ce^{-Rt/L}$ **35.** Proof

37. (a) $Q = 25e^{-t/20}$ (b) $-20\ln\left(\frac{3}{5}\right) \approx 10.2$ min (c) 0

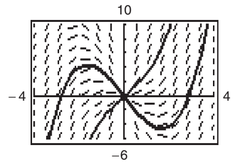
39. Answer (a) **41.** $\dfrac{dy}{dx} + P(x)y = Q(x)$; $u(x) = e^{\int P(x)\,dx}$

43. c **44.** d **45.** a **46.** b

47. (a)

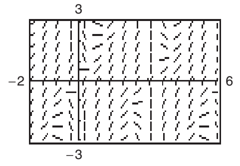


(c)

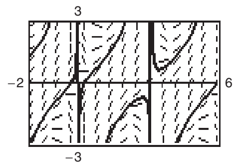


(b) $(-2, 4)$: $y = \frac{1}{2}x(x^2 - 8)$
$(2, 8)$: $y = \frac{1}{2}x(x^2 + 4)$

49. (a)



(b) $(1, 1)$: $y = (2\cos 1 + \sin 1)\csc x - 2\cot x$
$(3, -1)$: $y = (2\cos 3 - \sin 3)\csc x - 2\cot x$
(c)



51. $2e^x + e^{-2y} = C$ **53.** $y = Ce^{-\sin x} + 1$

55. $y = [e^x(x - 1) + C]/x^2$ **57.** $y = \frac{12}{5}x^2 + C/x^3$

59. $1/y^2 = Ce^{2x^3} + \frac{1}{3}$ **61.** $y = 1/(Cx - x^2)$

63. $1/y^2 = 2x + Cx^2$ **65.** $y^{2/3} = 2e^x + Ce^{2x/3}$

67. False. $y' + xy = x^2$ is linear.

Review Exercises for Chapter 6 *(page 431)*

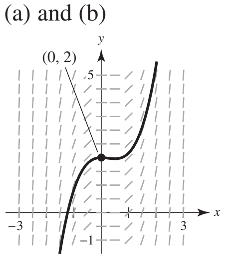
1. Yes **3.** $y = \frac{4}{3}x^3 + 7x + C$ **5.** $y = \frac{1}{2}\sin 2x + C$

7. $y = -e^{2-x} + C$

9.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	-10	-4	-4	0	2	8

11. (a) and (b)



13.

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
y_n	4	3.8	3.6125	3.4369	3.2726	3.1190	2.9756	2.8418	2.7172	2.6038	2.4986

15. $y = -\frac{5}{3}x^3 + x^2 + C$

17. $y = -3 - 1/(x + C)$ **19.** $y = Ce^x/(2 + x)^2$

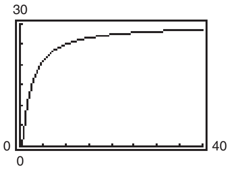
21. $\dfrac{dy}{dt} = \dfrac{k}{t^3}$; $y = -\dfrac{k}{2t^2} + C$ **23.** $y \approx \frac{3}{4}e^{0.379t}$

25. $y = \dfrac{9}{20}e^{(1/2)\ln(10/3)t}$ **27.** About 7.79 in.

29. About 37.5 yr

31. (a) $S \approx 30e^{-1.7918/t}$ (b) 20,965 units

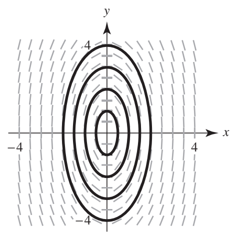
(c)



33. $y^2 = 5x^2 + C$ **35.** $y = Ce^{8x^2}$

37. $y^4 = 6x^2 - 8$ **39.** $y^4 = 2x^4 + 1$

41.



Graphs will vary.
$4x^2 + y^2 = C$

43. (a) 0.55 (b) 5250 (c) 150 (d) 6.41 yr

(e) $\dfrac{dP}{dt} = 0.55P\left(1 - \dfrac{P}{5250}\right)$

45. $y = \dfrac{80}{1 + 9e^{-t}}$

47. (a) $P(t) = \dfrac{20{,}400}{1 + 16e^{-0.553t}}$ (b) 17,118 trout (c) 4.94 yr

49. $y = -10 + Ce^x$ **51.** $y = e^{x/4}\left(\frac{1}{4}x + C\right)$

53. $y = (x + C)/(x - 2)$ **55.** $y = \frac{1}{10}e^{5x} + \frac{29}{10}e^{-5x}$

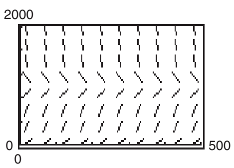
P.S. Problem Solving *(page 433)*

1. (a) $y = 1/(1 - 0.01t)^{100}$; $T = 100$

(b) $y = 1\Big/\left[\left(\dfrac{1}{y_0}\right)^{\varepsilon} - k\varepsilon t\right]^{1/\varepsilon}$; Explanations will vary.

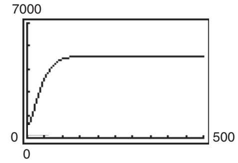
3. (a) $y = Le^{-Ce^{-kt}}$

(b)



(c) As $t \to \infty$, $y \to L$, the carrying capacity.

(d) $y_0 = 500 = 5000e^{-C} \implies e^C = 10 \implies C = \ln 10$



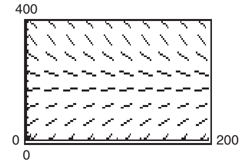
The graph is concave upward on $(0, 41.7)$ and downward on $(41.7, \infty)$.

5. 1481.45 sec $\approx$ 24 min, 41 sec

7. 2575.95 sec $\approx$ 42 min, 56 sec

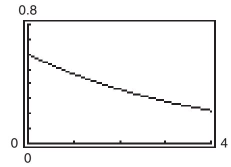
9. (a) $s = 184.21 - Ce^{-0.019t}$

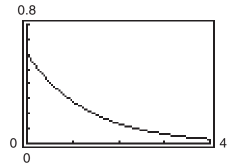
(b)



(c) As $t \to \infty$, $Ce^{-0.019t} \to 0$, and $s \to 184.21$.

11. (a) $C = 0.6e^{-0.25t}$ (b) $C = 0.6e^{-0.75t}$





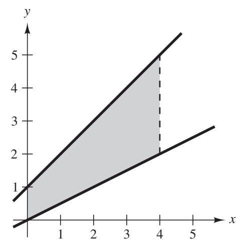
Chapter 7

Section 7.1 *(page 442)*

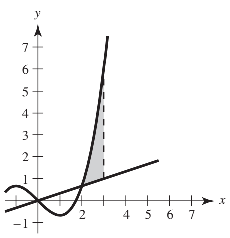
1. $-\displaystyle\int_0^6 (x^2 - 6x)\,dx$ **3.** $\displaystyle\int_0^3 (-2x^2 + 6x)\,dx$

5. $-6\displaystyle\int_0^1 (x^3 - x)\,dx$

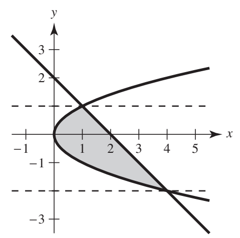
7.



9.



11.

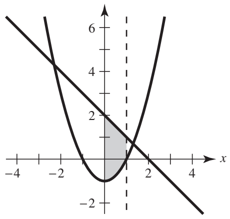


13. d

15. (a) $\frac{125}{6}$ (b) $\frac{125}{6}$

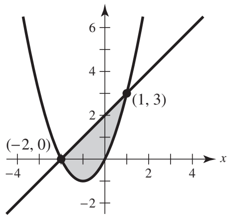
(c) Integrating with respect to y; Answers will vary.

17.



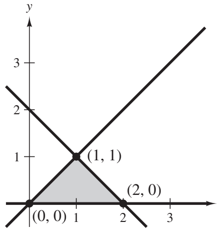
$\frac{32}{3}$

19.



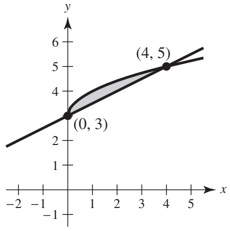
$\frac{9}{2}$

21.



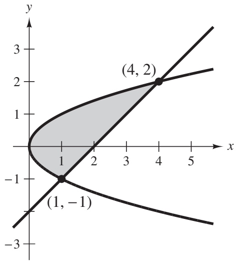
1

23.



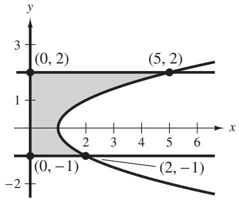
$\frac{4}{3}$

25.



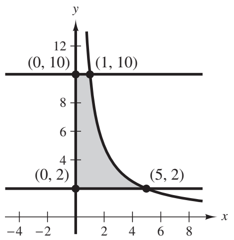
$\frac{9}{2}$

27.



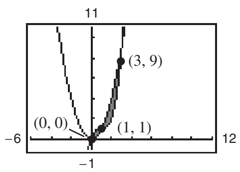
6

29.



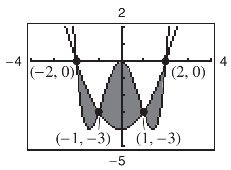
$10 \ln 5 \approx 16.094$

31. (a)



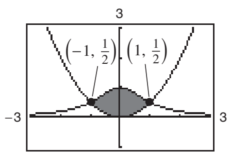
(b) $\frac{37}{12}$

33. (a)



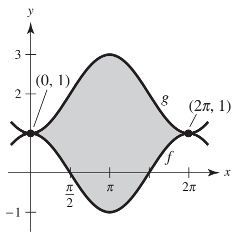
(b) 8

35. (a)



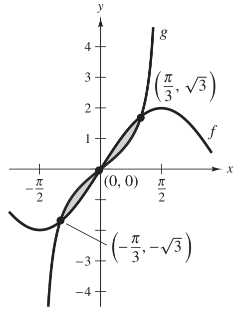
(b) $\pi/2 - 1/3 \approx 1.237$

37. (a)



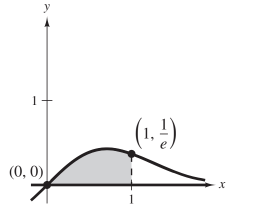
$4\pi \approx 12.566$

39.



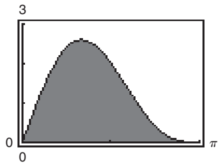
$2(1 - \ln 2) \approx 0.614$

41.



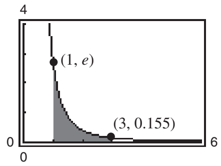
$(1/2)(1 - 1/e) \approx 0.316$

43. (a)



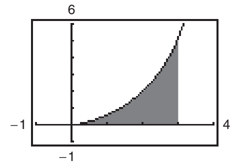
(b) 4

45. (a)



(b) About 1.323

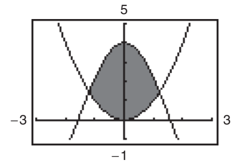
47. (a)



(b) The function is difficult to integrate.

(c) About 4.7721

49. (a)

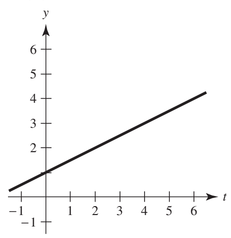


(b) The intersections are difficult to find.

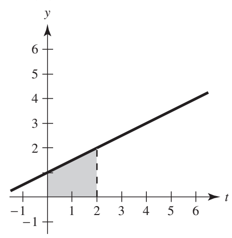
(c) About 6.3043

51. $F(x) = \frac{1}{4}x^2 + x$

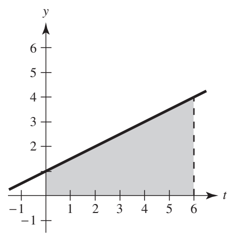
(a) $F(0) = 0$



(b) $F(2) = 3$

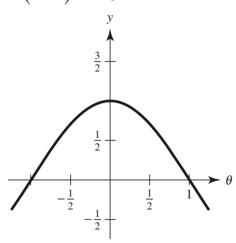


(c) $F(6) = 15$

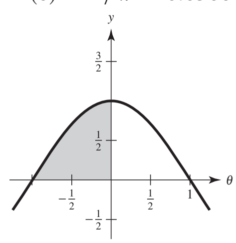


53. $F(\alpha) = (2/\pi)[\sin(\pi\alpha/2) + 1]$

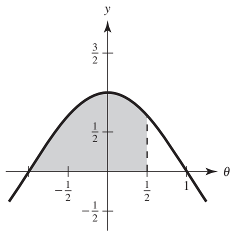
(a) $F(-1) = 0$



(b) $F(0) = 2/\pi \approx 0.6366$



(c) $F(1/2) = \left(\sqrt{2} + 2\right)/\pi \approx 1.0868$



55. 14 **57.** 16

59. Answers will vary. Sample answers:
(a) About 966 ft^2 (b) About 1004 ft^2

61. $\int_{-2}^{1} [x^3 - (3x - 2)]\, dx = \frac{27}{4}$

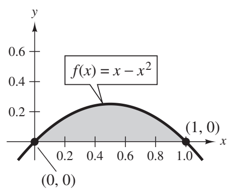
63. $\int_0^1 \left[\frac{1}{x^2+1} - \left(-\frac{1}{2}x + 1\right)\right] dx \approx 0.0354$

65. Answers will vary.
Example: $x^4 - 2x^2 + 1 \le 1 - x^2$ on $[-1, 1]$
$\int_{-1}^{1} [(1 - x^2) - (x^4 - 2x^2 + 1)]\, dx = \frac{4}{15}$

67. (a) The integral $\int_0^5 [v_1(t) - v_2(t)]\, dt = 10$ means that the first car traveled 10 more meters than the second car between 0 and 5 seconds.
The integral $\int_0^{10} [v_1(t) - v_2(t)]\, dt = 30$ means that the first car traveled 30 more meters than the second car between 0 and 10 seconds.
The integral $\int_{20}^{30} [v_1(t) - v_2(t)]\, dt = -5$ means that the second car traveled 5 more meters than the first car between 20 and 30 seconds.
(b) No. You do not know when both cars started or the initial distance between the cars.
(c) The car with velocity v_1 is ahead by 30 meters.
(d) Car 1 is ahead by 8 meters.

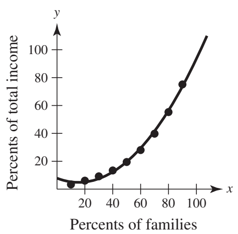
69. $b = 9\left(1 - 1/\sqrt[3]{4}\right) \approx 3.330$ **71.** $a = 4 - 2\sqrt{2} \approx 1.172$

73. Answers will vary. Sample answer: $\frac{1}{6}$

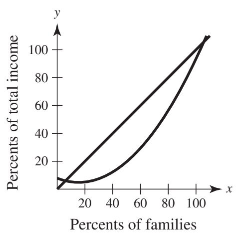


75. R_1; \$11.375 billion

77. (a) $y = 0.0124x^2 - 0.385x + 7.85$
(b)



(c)



(d) About 2006.7

79. (a) About 6.031 m^2 (b) About 12.062 m^3 (c) 60,310 lb

81. $\sqrt{3}/2 + 7\pi/24 + 1 \approx 2.7823$ **83.** True

85. False. Let $f(x) = x$ and $g(x) = 2x - x^2$. f and g intersect at $(1, 1)$, the midpoint of $[0, 2]$, but
$\int_a^b [f(x) - g(x)]\, dx = \int_0^2 [x - (2x - x^2)]\, dx = \frac{2}{3} \ne 0.$

87. Putnam Problem A1, 1993

Section 7.2 *(page 453)*

1. $\pi \int_0^1 (-x + 1)^2\, dx = \frac{\pi}{3}$ **3.** $\pi \int_1^4 \left(\sqrt{x}\right)^2 dx = \frac{15\pi}{2}$

5. $\pi \int_0^1 [(x^2)^2 - (x^5)^2]\, dx = \frac{6\pi}{55}$ **7.** $\pi \int_0^4 \left(\sqrt{y}\right)^2 dy = 8\pi$

9. $\pi \int_0^1 (y^{3/2})^2\, dy = \frac{\pi}{4}$

11. (a) $9\pi/2$ (b) $\left(36\pi\sqrt{3}\right)/5$ (c) $\left(24\pi\sqrt{3}\right)/5$
(d) $\left(84\pi\sqrt{3}\right)/5$

13. (a) $32\pi/3$ (b) $64\pi/3$ **15.** 18π

17. $\pi\left(48 \ln 2 - \frac{27}{4}\right) \approx 83.318$ **19.** $124\pi/3$

21. $832\pi/15$ **23.** $\pi \ln 5$ **25.** $2\pi/3$

27. $(\pi/2)(1 - 1/e^2) \approx 1.358$ **29.** $277\pi/3$ **31.** 8π

33. $\pi^2/2 \approx 4.935$ **35.** $(\pi/2)(e^2 - 1) \approx 10.036$

37. 1.969 **39.** 15.4115 **41.** $\pi/3$ **43.** $2\pi/15$

45. $\pi/2$ **47.** $\pi/6$

49. A sine curve on $[0, \pi/2]$ revolved about the x-axis

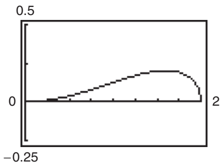
51. The parabola $y = 4x - x^2$ is a horizontal translation of the parabola $y = 4 - x^2$. Therefore, their volumes are equal.

53. (a) This statement is true. Explanations will vary.
(b) This statement is false. Explanations will vary.

55. $2\sqrt{2}$ **57.** $V = \frac{4}{3}\pi(R^2 - r^2)^{3/2}$ **59.** Proof

61. $\pi r^2 h[1 - (h/H) + h^2/(3H^2)]$

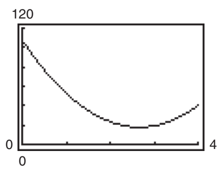
63.



$\pi/30$

65. (a) 60π (b) 50π

67. (a) $V = \pi\left(4b^2 - \frac{64}{3}b + \frac{512}{15}\right)$
(b)



(c) $b = \frac{8}{3} \approx 2.67$

$b \approx 2.67$

69. (a) ii; right circular cylinder of radius r and height h
(b) iv; ellipsoid whose underlying ellipse has the equation $(x/b)^2 + (y/a)^2 = 1$
(c) iii, sphere of radius r
(d) i; right circular cone of radius r and height h
(e) v; torus of cross-sectional radius r and other radius R

71. (a) $\frac{81}{10}$ (b) $\frac{9}{2}$ **73.** $\frac{16}{3}r^3$

75. (a) $\frac{2}{3}r^3$ (b) $\frac{2}{3}r^3 \tan\theta$; As $\theta \to 90°$, $V \to \infty$.

Section 7.3 *(page 462)*

1. $2\pi\int_0^2 x^2\,dx = \frac{16\pi}{3}$ **3.** $2\pi\int_0^4 x\sqrt{x}\,dx = \frac{128\pi}{5}$

5. $2\pi\int_0^4 \frac{1}{4}x^3\,dx = 32\pi$ **7.** $2\pi\int_0^2 x(4x - 2x^2)\,dx = \frac{16\pi}{3}$

9. $2\pi\int_0^2 x(x^2 - 4x + 4)\,dx = \frac{8\pi}{3}$

11. $2\pi\int_2^4 x\sqrt{x - 2}\,dx = \frac{128\pi}{15}\sqrt{2}$

13. $2\pi\int_0^1 x\left(\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\right)dx = \sqrt{2\pi}\left(1 - \frac{1}{\sqrt{e}}\right) \approx 0.986$

15. $2\pi\int_0^2 y(2 - y)\,dy = \frac{8\pi}{3}$

17. $2\pi\left[\int_0^{1/2} y\,dy + \int_{1/2}^1 y\left(\frac{1}{y} - 1\right)dy\right] = \frac{\pi}{2}$

19. $2\pi\int_0^8 y^{4/3}\,dy = \frac{768\pi}{7}$

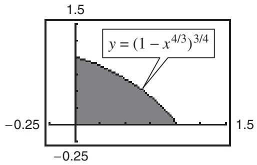
21. $2\pi\int_0^2 y(4 - 2y)\,dy = 16\pi/3$ **23.** 8π **25.** 16π

27. Shell method; it is much easier to put x in terms of y rather than vice versa.

29. (a) $128\pi/7$ (b) $64\pi/5$ (c) $96\pi/5$

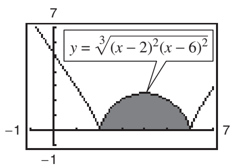
31. (a) $\pi a^3/15$ (b) $\pi a^3/15$ (c) $4\pi a^3/15$

33. (a)



(b) 1.506

35. (a)



(b) 187.25

37. (a) The rectangles would be vertical.
(b) The rectangles would be horizontal.

39. Both integrals yield the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x - 1}$, $y = 0$, and $x = 5$ about the x-axis.

41. a, c, b

43. (a) Region bounded by $y = x^2$, $y = 0$, $x = 0$, $x = 2$
(b) Revolved about the y-axis

45. (a) Region bounded by $x = \sqrt{6 - y}$, $y = 0$, $x = 0$
(b) Revolved about $y = -2$

47. Diameter $= 2\sqrt{4 - 2\sqrt{3}} \approx 1.464$ **49.** $4\pi^2$

51. (a) Proof (b) (i) $V = 2\pi$ (ii) $V = 6\pi^2$ **53.** Proof

55. (a) $R_1(n) = n/(n + 1)$ (b) $\lim_{n\to\infty} R_1(n) = 1$
(c) $V = \pi ab^{n+2}[n/(n + 2)]$; $R_2(n) = n/(n + 2)$
(d) $\lim_{n\to\infty} R_2(n) = 1$
(e) As $n \to \infty$, the graph approaches the line $x = b$.

57. (a) and (b) About 121,475 ft^3 **59.** $c = 2$

61. (a) $64\pi/3$ (b) $2048\pi/35$ (c) $8192\pi/105$

Section 7.4 *(page 473)*

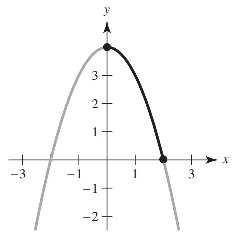
1. (a) and (b) 17 **3.** $\frac{5}{3}$ **5.** $\frac{2}{3}(2\sqrt{2} - 1) \approx 1.219$

7. $5\sqrt{5} - 2\sqrt{2} \approx 8.352$ **9.** 309.3195

11. $\ln[(\sqrt{2} + 1)/(\sqrt{2} - 1)] \approx 1.763$

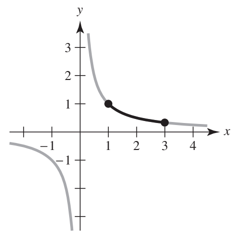
13. $\frac{1}{2}(e^2 - 1/e^2) \approx 3.627$ **15.** $\frac{76}{3}$

17. (a)



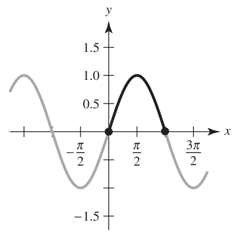
(b) $\int_0^2 \sqrt{1 + 4x^2}\,dx$
(c) About 4.647

19. (a)



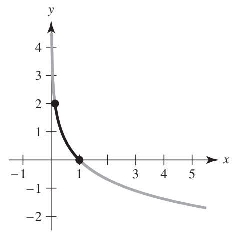
(b) $\int_1^3 \sqrt{1 + \frac{1}{x^4}}\,dx$
(c) About 2.147

21. (a)



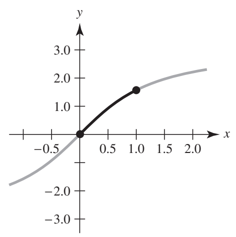
(b) $\int_0^{\pi} \sqrt{1 + \cos^2 x}\,dx$
(c) About 3.820

23. (a)



(b) $\int_0^2 \sqrt{1 + e^{-2y}}\,dy$
$= \int_{e^{-2}}^1 \sqrt{1 + \frac{1}{x^2}}\,dx$
(c) About 2.221

25. (a)



(b) $\int_0^1 \sqrt{1 + \left(\frac{2}{1 + x^2}\right)^2}\,dx$
(c) About 1.871

27. b

29. (a) 64.125 (b) 64.525 (c) 64.666 (d) 64.672

31. $20[\sinh 1 - \sinh(-1)] \approx 47.0$ m **33.** About 1480

35. $3\arcsin\frac{2}{3} \approx 2.1892$

37. $2\pi\int_0^3 \frac{1}{3}x^3\sqrt{1 + x^4}\,dx = \frac{\pi}{9}(82\sqrt{82} - 1) \approx 258.85$

39. $2\pi\int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)dx = \frac{47\pi}{16} \approx 9.23$

41. $2\pi\int_{-1}^1 2\,dx = 8\pi \approx 25.13$

43. $2\pi\int_1^8 x\sqrt{1 + \frac{1}{9x^{4/3}}}\,dx = \frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10}) \approx 199.48$

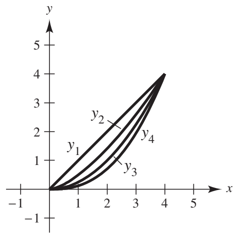
45. $2\pi\int_0^2 x\sqrt{1 + \frac{x^2}{4}}\,dx = \frac{\pi}{3}(16\sqrt{2} - 8) \approx 15.318$

47. 14.424

49. A rectifiable curve is a curve with a finite arc length.

51. The integral formula for the area of a surface of revolution is derived from the formula for the lateral surface area of the frustum of a right circular cone. The formula is $S = 2\pi rL$, where $r = \frac{1}{2}(r_1 + r_2)$, which is the average radius of the frustum, and L is the length of a line segment on the frustum. The representative element is $2\pi f(d_i)\sqrt{1 + (\Delta y_i/\Delta x_i)^2}\,\Delta x_i$.

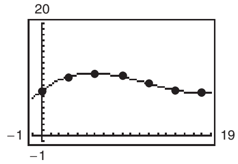
53. (a)



(b) y_1, y_2, y_3, y_4
(c) $s_1 \approx 5.657$; $s_2 \approx 5.759$; $s_3 \approx 5.916$; $s_4 \approx 6.063$

55. 20π **57.** $6\pi(3 - \sqrt{5}) \approx 14.40$

59. (a) Answers will vary. Sample answer: 5207.62 in.^{3}
(b) Answers will vary. Sample answer: 1168.64 in.^{2}
(c) $r = 0.0040y^3 - 0.142y^2 + 1.23y + 7.9$



(d) 5279.64 in.^{3}; 1179.5 in.^{2}

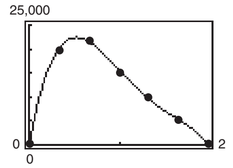
61. (a) $\pi(1 - 1/b)$ (b) $2\pi\int_1^b \sqrt{x^4+1}/x^3\,dx$
(c) $\lim_{b\to\infty} V = \lim_{b\to\infty} \pi(1 - 1/b) = \pi$
(d) Because $\dfrac{\sqrt{x^4+1}}{x^3} > \dfrac{\sqrt{x^4}}{x^3} = \dfrac{1}{x} > 0$ on $[1, b]$,
you have $\displaystyle\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx > \int_1^b \frac{1}{x}\,dx = \Big[\ln x\Big]_1^b = \ln b$
and $\lim_{b\to\infty} \ln b \to \infty$. So, $\displaystyle\lim_{b\to\infty} 2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx = \infty$.

63. Fleeing object: $\frac{2}{3}$ unit
Pursuer: $\displaystyle\frac{1}{2}\int_0^1 \frac{x+1}{\sqrt{x}}\,dx = \frac{4}{3} = 2\left(\frac{2}{3}\right)$

65. $384\pi/5$ **67–69.** Proofs

Section 7.5 *(page 483)*

1. 48,000 ft-lb **3.** 896 N-m **5.** 40.833 in.-lb $\approx$ 3.403 ft-lb
7. 160 in.-lb $\approx$ 13.3 ft-lb **9.** 37.125 ft-lb
11. (a) 487.805 mile-tons $\approx 5.151 \times 10^9$ ft-lb
(b) 1395.349 mile-tons $\approx 1.473 \times 10^{10}$ ft-lb
13. (a) 2.93×10^4 mile-tons $\approx 3.10 \times 10^{11}$ ft-lb
(b) 3.38×10^4 mile-tons $\approx 3.57 \times 10^{11}$ ft-lb
15. (a) 2496 ft-lb (b) 9984 ft-lb **17.** $470{,}400\pi$ N-m
19. 2995.2π ft-lb **21.** $20{,}217.6\pi$ ft-lb **23.** 2457π ft-lb
25. 600 ft-lb **27.** 450 ft-lb **29.** 168.75 ft-lb
31. If an object is moved a distance D in the direction of an applied constant force F, then the work W done by the force is defined as $W = FD$.
33. The situation in part (a) requires more work. There is no work required for part (b) because the distance is 0.
35. (a) 54 ft-lb (b) 160 ft-lb (c) 9 ft-lb (d) 18 ft-lb
37. $2000 \ln(3/2) \approx 810.93$ ft-lb **39.** 3249.4 ft-lb
41. 10,330.3 ft-lb
43. (a) $16{,}000\pi$ ft-lb (b) 24,888.889 ft-lb
(c) $F(x) = -16{,}261.36x^4 + 85{,}295.45x^3 - 157{,}738.64x^2 + 104{,}386.36x - 32.4675$

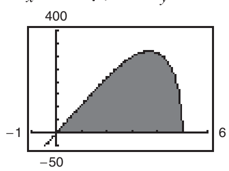


(d) 0.524 ft (e) 25,180.5 ft-lb

Section 7.6 *(page 494)*

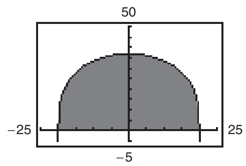
1. $\bar{x} = -\frac{4}{3}$ **3.** $\bar{x} = 4$ **5.** (a) $\bar{x} = 8$ (b) $\bar{x} = -\frac{3}{4}$
7. $x = 6$ ft **9.** $(\bar{x}, \bar{y}) = \left(\frac{10}{9}, -\frac{1}{9}\right)$ **11.** $(\bar{x}, \bar{y}) = \left(2, \frac{48}{25}\right)$
13. $M_x = \rho/3, M_y = 4\rho/3, \quad (\bar{x}, \bar{y}) = (4/3, 1/3)$
15. $M_x = 4\rho, M_y = 64\rho/5, \quad (\bar{x}, \bar{y}) = (12/5, 3/4)$
17. $M_x = \rho/35, M_y = \rho/20, \quad (\bar{x}, \bar{y}) = (3/5, 12/35)$
19. $M_x = 99\rho/5, M_y = 27\rho/4, \quad (\bar{x}, \bar{y}) = (3/2, 22/5)$
21. $M_x = 192\rho/7, M_y = 96\rho, \quad (\bar{x}, \bar{y}) = (5, 10/7)$
23. $M_x = 0, M_y = 256\rho/15, \quad (\bar{x}, \bar{y}) = (8/5, 0)$
25. $M_x = 27\rho/4, M_y = -27\rho/10, \quad (\bar{x}, \bar{y}) = (-3/5, 3/2)$

27.



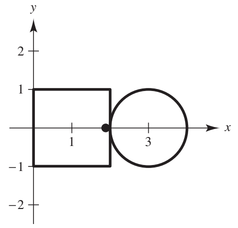
$(\bar{x}, \bar{y}) = (3.0, 126.0)$

29.



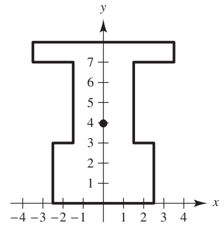
$(\bar{x}, \bar{y}) = (0, 16.2)$

31.



$(\bar{x}, \bar{y}) = \left(\dfrac{4 + 3\pi}{4 + \pi}, 0\right)$

33.

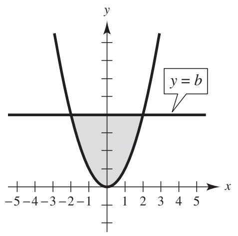


$(\bar{x}, \bar{y}) = \left(0, \dfrac{135}{34}\right)$

35. $(\bar{x}, \bar{y}) = \left(\dfrac{2 + 3\pi}{2 + \pi}, 0\right)$ **37.** $160\pi^2 \approx 1579.14$
39. $128\pi/3 \approx 134.04$
41. The center of mass $(\bar{x}, \bar{y})$ is $\bar{x} = M_y/m$ and $\bar{y} = M_x/m$, where:
1. $m = m_1 + m_2 + \cdot\cdot\cdot + m_n$ is the total mass of the system.
2. $M_y = m_1x_1 + m_2x_2 + \cdot\cdot\cdot + m_nx_n$ is the moment about the y-axis.
3. $M_x = m_1y_1 + m_2y_2 + \cdot\cdot\cdot + m_ny_n$ is the moment about the x-axis.

43. See Theorem 7.1 on page 493. **45.** $(\bar{x}, \bar{y}) = \left(\dfrac{b}{3}, \dfrac{c}{3}\right)$
47. $(\bar{x}, \bar{y}) = \left(\dfrac{(a + 2b)c}{3(a + b)}, \dfrac{a^2 + ab + b^2}{3(a + b)}\right)$
49. $(\bar{x}, \bar{y}) = (0, 4b/(3\pi))$

51. (a)



(b) $\bar{x} = 0$ by symmetry

(c) $M_y = \int_{-\sqrt{b}}^{\sqrt{b}} x(b - x^2)\, dx = 0$ because $x(b - x^2)$ is an odd function.

(d) $\bar{y} > b/2$ because the area is greater for $y > b/2$.

(e) $\bar{y} = (3/5)b$

53. (a) $(\bar{x}, \bar{y}) = (0, 12.98)$

(b) $y = (-1.02 \times 10^{-5})x^4 - 0.0019x^2 + 29.28$

(c) $(\bar{x}, \bar{y}) = (0, 12.85)$

55. $(\bar{x}, \bar{y}) = (0, 2r/\pi)$

57. $(\bar{x}, \bar{y}) = \left(\dfrac{n+1}{n+2}, \dfrac{n+1}{4n+2}\right)$; As $n \to \infty$, the region shrinks toward the line segments $y = 0$ for $0 \le x \le 1$ and $x = 1$ for $0 \le y \le 1$; $(\bar{x}, \bar{y}) \to \left(1, \dfrac{1}{4}\right)$.

Section 7.7 *(page 501)*

1. 1497.6 lb **3.** 4992 lb **5.** 748.8 lb **7.** 1123.2 lb

9. 748.8 lb **11.** 1064.96 lb **13.** 117,600 N

15. 2,381,400 N **17.** 2814 lb **19.** 6753.6 lb

21. 94.5 lb **23–25.** Proofs **27.** 960 lb

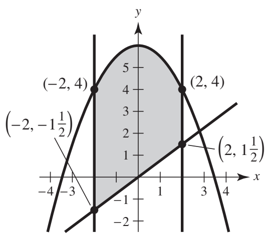
29. Answers will vary. Sample answer (using Simpson's Rule): 3010.8 lb

31. $3\sqrt{2}/2 \approx 2.12$ ft; The pressure increases with increasing depth.

33. Because you are measuring total force against a region between two depths

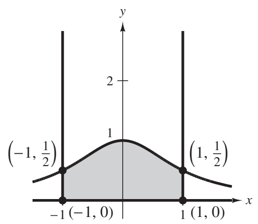
Review Exercises for Chapter 7 *(page 503)*

1.



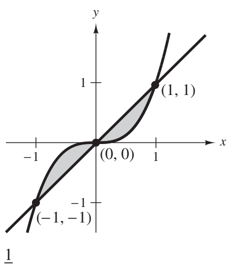
$64/3$

3.



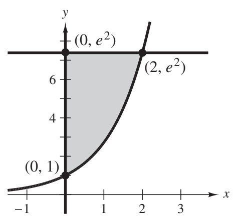
$\pi/2$

5.



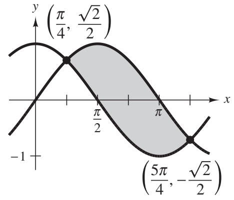
$\frac{1}{2}$

7.



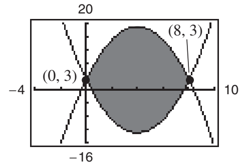
$e^2 + 1$

9.



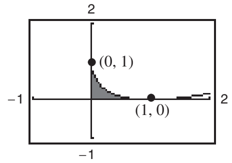
$2\sqrt{2}$

11.



$\frac{512}{3}$

13.



$\frac{1}{6}$

15. (a) 9920 ft^2 (b) $10{,}413\frac{1}{3}$ ft^2

17. (a) 9π (b) 18π (c) 9π (d) 36π **19.** $\pi^2/4$

21. $2\pi \ln 2.5 \approx 5.757$ **23.** 1.958 ft

25. $\frac{8}{15}(1 + 6\sqrt{3}) \approx 6.076$ **27.** 4018.2 ft **29.** 15π

31. 62.5 in.-lb $\approx$ 5.208 ft-lb

33. $122{,}980\pi$ ft-lb $\approx$ 193.2 foot-tons **35.** 200 ft-lb

37. $a = 15/4$ **39.** 3.6 **41.** $(\bar{x}, \bar{y}) = \left(1, \frac{17}{5}\right)$

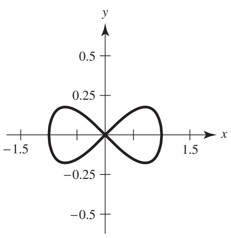
43. $(\bar{x}, \bar{y}) = \left(\dfrac{2(9\pi + 49)}{3(\pi + 9)}, 0\right)$ **45.** 3072 lb

47. Wall at shallow end: 15,600 lb
Wall at deep end: 62,400 lb
Side wall: 72,800 lb

P.S. Problem Solving *(page 505)*

1. 3 **3.** $y = 0.2063x$

5.

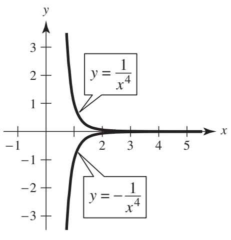


$\dfrac{5\sqrt{2}\pi}{3}$

7. $V = 2\pi\left[d + \frac{1}{2}\sqrt{w^2 + l^2}\right]lw$

9. $f(x) = 2e^{x/2} - 2$ **11.** 89.3%

13.



(a) $(\bar{x}, \bar{y}) = \left(\dfrac{63}{43}, 0\right)$

(b) $(\bar{x}, \bar{y}) = \left(\dfrac{3b(b+1)}{2(b^2 + b + 1)}, 0\right)$

(c) $\left(\dfrac{3}{2}, 0\right)$

15. Consumer surplus: 1600; Producer surplus: 400

17. Wall at shallow end: 9984 lb
Wall at deep end: 39,936 lb
Side wall: 19,968 + 26,624 = 46,592 lb

Chapter 8

Section 8.1 *(page 512)*

1. b **3.** c

5. $\int u^n\,du$, $u = 5x - 3, n = 4$ **7.** $\int \frac{du}{u}$, $u = 1 - 2\sqrt{x}$ **9.** $\int \frac{du}{\sqrt{a^2 - u^2}}$, $u = t, a = 1$

11. $\int \sin u\,du$, $u = t^2$ **13.** $\int e^u\,du$, $u = \sin x$ **15.** $2(x - 5)^7 + C$

17. $-7/[6(z - 10)^6] + C$ **19.** $\frac{1}{2}v^2 - 1/[6(3v - 1)^2] + C$

21. $-\frac{1}{3}\ln|-t^3 + 9t + 1| + C$

23. $\frac{1}{2}x^2 + x + \ln|x - 1| + C$ **25.** $\ln(1 + e^x) + C$

27. $\frac{x}{15}(48x^4 + 200x^2 + 375) + C$ **29.** $\sin(2\pi x^2)/(4\pi) + C$

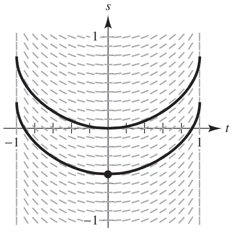
31. $-2\sqrt{\cos x} + C$ **33.** $2\ln(1 + e^x) + C$

35. $(\ln x)^2 + C$ **37.** $-\ln|\csc\alpha + \cot\alpha| + \ln|\sin\alpha| + C$

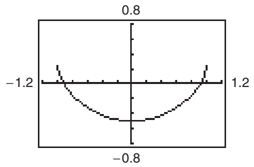
39. $-\frac{1}{4}\arcsin(4t + 1) + C$ **41.** $\frac{1}{2}\ln|\cos(2/t)| + C$

43. $6\arcsin[(x - 5)/5] + C$ **45.** $\frac{1}{4}\arctan[(2x + 1)/8] + C$

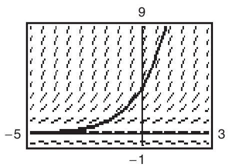
47. (a)



(b) $\frac{1}{2}\arcsin t^2 - \frac{1}{2}$



49. $y = 4e^{0.8x}$



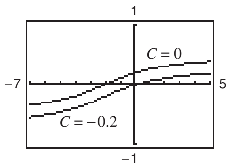
51. $y = \frac{1}{2}e^{2x} + 10e^x + 25x + C$ **53.** $r = 10\arcsin e^t + C$

55. $y = \frac{1}{2}\arctan(\tan x/2) + C$ **57.** $\frac{1}{2}$

59. $\frac{1}{2}(1 - e^{-1}) \approx 0.316$ **61.** 8 **63.** $\pi/18$

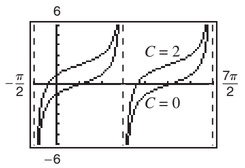
65. $18\sqrt{6}/5 \approx 8.82$ **67.** $\frac{4}{3} \approx 1.333$

69. $\frac{1}{3}\arctan\left[\frac{1}{3}(x + 2)\right] + C$
Graphs will vary.
Example:



One graph is a vertical translation of the other.

71. $\tan\theta - \sec\theta + C$
Graphs will vary.
Example:



One graph is a vertical translation of the other.

73. Power Rule: $\int u^n\,du = \frac{u^{n+1}}{n + 1} + C$; $u = x^2 + 1, n = 3$

75. Log Rule: $\int \frac{du}{u} = \ln|u| + C$; $u = x^2 + 1$

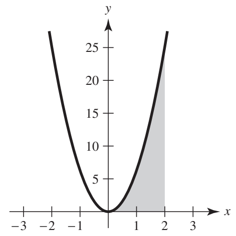
77. $a = \sqrt{2}, b = \frac{\pi}{4}$; $-\frac{1}{\sqrt{2}}\ln\left|\csc\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right| + C$

79. $a = \frac{1}{2}$

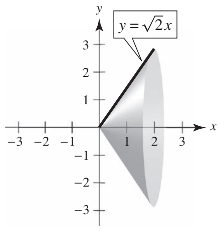
81. (a) They are equivalent because $e^{x + C_1} = e^x \cdot e^{C_1} = Ce^x, C = e^{C_1}$.
(b) They differ by a constant.
$\sec^2 x + C_1 = (\tan^2 x + 1) + C_1 = \tan^2 x + C$

83. a

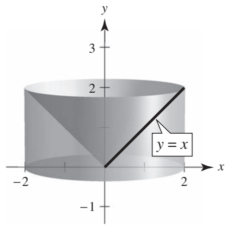
85. (a)



(b)



(c)



87. (a) $\pi(1 - e^{-1}) \approx 1.986$
(b) $b = \sqrt{\ln\left(\frac{3\pi}{3\pi - 4}\right)} \approx 0.743$

89. $\ln(\sqrt{2} + 1) \approx 0.8814$

91. $(8\pi/3)(10\sqrt{10} - 1) \approx 256.545$ **93.** $\frac{1}{3}\arctan 3 \approx 0.416$

95. About 1.0320

97. (a) $\frac{1}{3}\sin x(\cos^2 x + 2)$
(b) $\frac{1}{15}\sin x(3\cos^4 x + 4\cos^2 x + 8)$
(c) $\frac{1}{35}\sin x(5\cos^6 x + 6\cos^4 x + 8\cos^2 x + 16)$
(d) $\int \cos^{15} x\,dx = \int (1 - \sin^2 x)^7 \cos x\,dx$
You would expand $(1 - \sin^2 x)^7$.

99. Proof

Section 8.2 *(page 521)*

1. $u = x, dv = e^{2x}\,dx$ **3.** $u = (\ln x)^2, dv = dx$

5. $u = x, dv = \sec^2 x\,dx$ **7.** $\frac{1}{16}x^4(4\ln x - 1) + C$

9. $\frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x + C$ **11.** $-\frac{1}{16e^{4x}}(4x + 1) + C$

13. $e^x(x^3 - 3x^2 + 6x - 6) + C$

15. $\frac{1}{4}[2(t^2 - 1)\ln|t + 1| - t^2 + 2t] + C$ **17.** $\frac{1}{3}(\ln x)^3 + C$

19. $e^{2x}/[4(2x + 1)] + C$ **21.** $\frac{2}{15}(x - 5)^{3/2}(3x + 10) + C$

23. $x\sin x + \cos x + C$

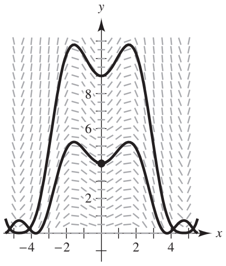
25. $(6x - x^3)\cos x + (3x^2 - 6)\sin x + C$

27. $x\arctan x - \frac{1}{2}\ln(1 + x^2) + C$

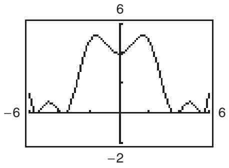
29. $-\frac{3}{34}e^{-3x}\sin 5x - \frac{5}{34}e^{-3x}\cos 5x + C$ **31.** $x\ln x - x + C$

33. $y = \frac{2}{5}t^2\sqrt{3 + 5t} - \frac{8t}{75}(3 + 5t)^{3/2} + \frac{16}{1875}(3 + 5t)^{5/2} + C$
$= \frac{2}{625}\sqrt{3 + 5t}(25t^2 - 20t + 24) + C$

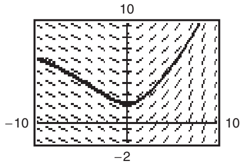
35. (a)



(b) $2\sqrt{y} - \cos x - x \sin x = 3$



37.



39. $2e^{3/2} + 4 \approx 12.963$

41. $\dfrac{\pi}{8} - \dfrac{1}{4} \approx 0.143$

43. $(\pi - 3\sqrt{3} + 6)/6 \approx 0.658$

45. $\frac{1}{2}[e(\sin 1 - \cos 1) + 1] \approx 0.909$

47. $8 \operatorname{arcsec} 4 + \sqrt{3}/2 - \sqrt{15}/2 - 2\pi/3 \approx 7.380$

49. $(e^{2x}/4)(2x^2 - 2x + 1) + C$

51. $(3x^2 - 6) \sin x - (x^3 - 6x) \cos x + C$

53. $x \tan x + \ln|\cos x| + C$

55. $2(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}) + C$

57. $\frac{1}{2}(x^4 e^{x^2} - 2x^2 e^{x^2} + 2e^{x^2}) + C$

59. (a) Product Rule
(b) Answers will vary. Sample answer: You want dv to be the most complicated portion of the integrand.

61. (a) No, substitution (b) Yes, $u = \ln x$, $dv = x\,dx$
(c) Yes, $u = x^2$, $dv = e^{-3x}\,dx$ (d) No, substitution
(e) Yes, $u = x$ and $dv = \dfrac{1}{\sqrt{x+1}}\,dx$ (f) No, substitution

63. $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

65. $n = 0$: $x(\ln x - 1) + C$
$n = 1$: $\frac{1}{4}x^2(2 \ln x - 1) + C$
$n = 2$: $\frac{1}{9}x^3(3 \ln x - 1) + C$
$n = 3$: $\frac{1}{16}x^4(4 \ln x - 1) + C$
$n = 4$: $\frac{1}{25}x^5(5 \ln x - 1) + C$

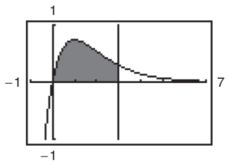
$$\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$$

67–71. Proofs **73.** $-x^2 \cos x + 2x \sin x + 2 \cos x + C$

75. $\frac{1}{36}x^6(6 \ln x - 1) + C$

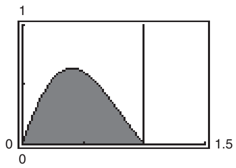
77. $\dfrac{e^{-3x}(-3 \sin 4x - 4 \cos 4x)}{25} + C$

79.



$2 - \dfrac{8}{e^3} \approx 1.602$

81.



$\dfrac{\pi}{1 + \pi^2}\left(\dfrac{1}{e} + 1\right) \approx 0.395$

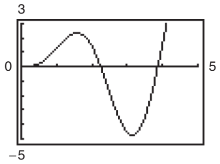
83. (a) 1 (b) $\pi(e - 2) \approx 2.257$ (c) $\frac{1}{2}\pi(e^2 + 1) \approx 13.177$
(d) $\left(\dfrac{e^2 + 1}{4}, \dfrac{e - 2}{2}\right) \approx (2.097, 0.359)$

85. In Example 6, we showed that the centroid of an equivalent region was $(1, \pi/8)$. By symmetry, the centroid of this region is $(\pi/8, 1)$.

87. $[7/(10\pi)](1 - e^{-4\pi}) \approx 0.223$ **89.** $931,265

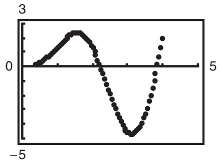
91. Proof **93.** $b_n = [8h/(n\pi)^2] \sin(n\pi/2)$

95. (a) $y = \frac{1}{4}(3 \sin 2x - 6x \cos 2x)$
(b)



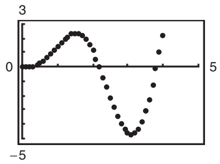
(c) You obtain the following points.

n	x_n	y_n
0	0	0
1	0.05	0
2	0.10	7.4875×10^{-4}
3	0.15	0.0037
4	0.20	0.0104
⋮	⋮	⋮
80	4.00	1.3181



(d) You obtain the following points.

n	x_n	y_n
0	5	0
1	0.1	0
2	0.2	0.0060
3	0.3	0.0293
4	0.4	0.0801
⋮	⋮	⋮
40	4.0	1.0210



97. The graph of $y = x \sin x$ is below the graph of $y = x$ on $[0, \pi/2]$.

99. For any integrable function, $\int f(x)\,dx = C + \int f(x)\,dx$, but this cannot be used to imply that $C = 0$.

Section 8.3 *(page 530)*

1. $-\frac{1}{6}\cos^6 x + C$ **3.** $\frac{1}{16}\sin^8 2x + C$

5. $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

7. $-\frac{1}{3}(\cos 2\theta)^{3/2} + \frac{1}{7}(\cos 2\theta)^{7/2} + C$

9. $\frac{1}{12}(6x + \sin 6x) + C$

11. $\frac{1}{8}(2x^2 - 2x \sin 2x - \cos 2x) + C$ **13.** $\frac{16}{35}$

15. $63\pi/512$ **17.** $5\pi/32$ **19.** $\frac{1}{4}\ln|\sec 4x + \tan 4x| + C$

21. $(\sec \pi x \tan \pi x + \ln|\sec \pi x + \tan \pi x|)/(2\pi) + C$

23. $\frac{1}{2}\tan^4(x/2) - \tan^2(x/2) - 2\ln|\cos(x/2)| + C$

25. $\dfrac{1}{2}\left[\dfrac{\sec^5 2t}{5} - \dfrac{\sec^3 2t}{3}\right] + C$ **27.** $\dfrac{1}{24}\sec^6 4x + C$

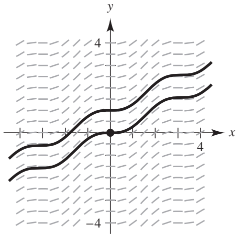
29. $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

31. $\ln|\sec x + \tan x| - \sin x + C$

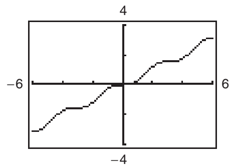
33. $(12\pi\theta - 8 \sin 2\pi\theta + \sin 4\pi\theta)/(32\pi) + C$

35. $y = \frac{1}{9}\sec^3 3x - \frac{1}{3}\sec 3x + C$

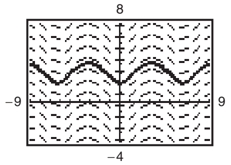
37. (a)



(b) $y = \frac{1}{2}x - \frac{1}{4}\sin 2x$



39.



41. $\frac{1}{16}(2\sin 4x + \sin 8x) + C$

43. $\frac{1}{12}(3\cos 2x - \cos 6x) + C$ **45.** $\frac{1}{8}(2\sin 2\theta - \sin 4\theta) + C$

47. $\frac{1}{4}\left(\ln|\csc^2 2x| - \cot^2 2x\right) + C$

49. $-\frac{1}{3}\cot 3x - \frac{1}{9}\cot^3 3x + C$

51. $\ln|\csc t - \cot t| + \cos t + C$

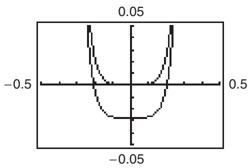
53. $\ln|\csc x - \cot x| + \cos x + C$ **55.** $t - 2\tan t + C$

57. π **59.** $3(1 - \ln 2)$ **61.** $\ln 2$ **63.** 4

65. (a) Save one sine factor and convert the remaining factors to cosines. Then expand and integrate.
(b) Save one cosine factor and convert the remaining factors to sines. Then expand and integrate.
(c) Make repeated use of the power reducing formulas to convert the integrand to odd powers of the cosine. Then proceed as in part (b).

67. (a) $\frac{1}{2}\sin^2 x + C$ (b) $-\frac{1}{2}\cos^2 x + C$
(c) $\frac{1}{2}\sin^2 x + C$ (d) $-\frac{1}{4}\cos 2x + C$
The answers are all the same; they are just written in different forms. Using trigonometric identities, you can rewrite each answer in the same form.

69. (a) $\frac{1}{18}\tan^6 3x + \frac{1}{12}\tan^4 3x + C_1$, $\frac{1}{18}\sec^6 3x - \frac{1}{12}\sec^4 3x + C_2$
(b)



(c) Proof

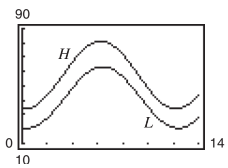
71. $\frac{1}{3}$ **73.** 1 **75.** $2\pi(1 - \pi/4) \approx 1.348$

77. (a) $\pi^2/2$ (b) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ **79–81.** Proofs

83. $-\frac{1}{15}\cos x(3\sin^4 x + 4\sin^2 x + 8) + C$

85. $\dfrac{5}{6\pi}\tan\dfrac{2\pi x}{5}\left(\sec^2\dfrac{2\pi x}{5} + 2\right) + C$

87. (a) $H(t) \approx 57.72 - 23.36\cos(\pi t/6) - 2.75\sin(\pi t/6)$
(b) $L(t) \approx 42.04 - 20.91\cos(\pi t/6) - 4.33\sin(\pi t/6)$
(c)



The maximum difference is at $t \approx 4.9$, or late spring.

89. Proof

Section 8.4 *(page 539)*

1. $x = 3\tan\theta$ **3.** $x = 5\sin\theta$ **5.** $x/\left(16\sqrt{16 - x^2}\right) + C$

7. $4\ln\left|\left(4 - \sqrt{16 - x^2}\right)/x\right| + \sqrt{16 - x^2} + C$

9. $\ln\left|x + \sqrt{x^2 - 25}\right| + C$

11. $\frac{1}{15}(x^2 - 25)^{3/2}(3x^2 + 50) + C$

13. $\frac{1}{3}(1 + x^2)^{3/2} + C$ **15.** $\frac{1}{2}[\arctan x + x/(1 + x^2)] + C$

17. $\frac{1}{2}x\sqrt{9 + 16x^2} + \frac{9}{8}\ln\left|4x + \sqrt{9 + 16x^2}\right| + C$

19. $\frac{25}{4}\arcsin(2x/5) + \frac{1}{2}x\sqrt{25 - 4x^2} + C$

21. $\arcsin(x/4) + C$ **23.** $4\arcsin(x/2) + x\sqrt{4 - x^2} + C$

25. $-\dfrac{(1 - x^2)^{3/2}}{3x^3} + C$ **27.** $-\dfrac{1}{3}\ln\left|\dfrac{\sqrt{4x^2 + 9} + 3}{2x}\right| + C$

29. $3/\sqrt{x^2 + 3} + C$

31. $\frac{1}{2}\left(\arcsin e^x + e^x\sqrt{1 - e^{2x}}\right) + C$

33. $\frac{1}{4}\left[x/(x^2 + 2) + \left(1/\sqrt{2}\right)\arctan\left(x/\sqrt{2}\right)\right] + C$

35. $x\,\text{arcsec}\,2x - \frac{1}{2}\ln\left|2x + \sqrt{4x^2 - 1}\right| + C$

37. $\arcsin[(x - 2)/2] + C$

39. $\sqrt{x^2 + 6x + 12} - 3\ln\left|\sqrt{x^2 + 6x + 12} + (x + 3)\right| + C$

41. (a) and (b) $\sqrt{3} - \pi/3 \approx 0.685$

43. (a) and (b) $9\left(2 - \sqrt{2}\right) \approx 5.272$

45. (a) and (b) $-(9/2)\ln\left(2\sqrt{7}/3 - 4\sqrt{3}/3 - \sqrt{21}/3 + 8/3\right) + 9\sqrt{3} - 2\sqrt{7} \approx 12.644$

47. (a) Let $u = a\sin\theta$, $\sqrt{a^2 - u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2$.
(b) Let $u = a\tan\theta$, $\sqrt{a^2 + u^2} = a\sec\theta$, where $-\pi/2 < \theta < \pi/2$.
(c) Let $u = a\sec\theta$, $\sqrt{u^2 - a^2} = \tan\theta$ if $u > a$ and $\sqrt{u^2 - a^2} = -\tan\theta$ if $u < -a$, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$.

49. (a) $\frac{1}{2}\ln(x^2 + 9) + C$; The answers are equivalent.
(b) $x - 3\arctan(x/3) + C$; The answers are equivalent.

51. True

53. False. $\displaystyle\int_0^{\sqrt{3}}\frac{dx}{(1 + x^2)^{3/2}} = \int_0^{\pi/3}\cos\theta\,d\theta$

55. πab

57. (a) $5\sqrt{2}$ (b) $25(1 - \pi/4)$ (c) $r^2(1 - \pi/4)$

59. $6\pi^2$ **61.** $\ln\left[\dfrac{5\left(\sqrt{2} + 1\right)}{\sqrt{26} + 1}\right] + \sqrt{26} - \sqrt{2} \approx 4.367$

63. Length of one arch of sine curve: $y = \sin x$, $y' = \cos x$

$$L_1 = \int_0^{\pi}\sqrt{1 + \cos^2 x}\,dx$$

Length of one arch of cosine curve: $y = \cos x$, $y' = -\sin x$

$$L_2 = \int_{-\pi/2}^{\pi/2}\sqrt{1 + \sin^2 x}\,dx$$

$$= \int_{-\pi/2}^{\pi/2}\sqrt{1 + \cos^2(x - \pi/2)}\,dx,\quad u = x - \pi/2,\ du = dx$$

$$= \int_{-\pi}^{0}\sqrt{1 + \cos^2 u}\,du = \int_0^{\pi}\sqrt{1 + \cos^2 u}\,du = L_1$$

65. $(0, 0.422)$

67. $(\pi/32)\left[102\sqrt{2} - \ln\left(3 + 2\sqrt{2}\right)\right] \approx 13.989$

69. (a) 187.2π lb (b) $62.4\pi d$ lb **71.** Proof

73. $12 + 9\pi/2 - 25\arcsin(3/5) \approx 10.050$

75. Putnum Problem A5, 2005

Section 8.5 *(page 549)*

1. $\dfrac{A}{x} + \dfrac{B}{x - 8}$ **3.** $\dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 10}$

5. $\frac{1}{6}\ln|(x - 3)/(x + 3)| + C$ **7.** $\ln|(x - 1)/(x + 4)| + C$

9. $5 \ln|x-2| - \ln|x+2| - 3\ln|x| + C$
11. $x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C$
13. $1/x + \ln|x^4 + x^3| + C$
15. $2\ln|x-2| - \ln|x| - 3/(x-2) + C$
17. $\ln|(x^2+1)/x| + C$
19. $\frac{1}{6}\left[\ln|(x-2)/(x+2)| + \sqrt{2}\arctan\left(x/\sqrt{2}\right)\right] + C$
21. $\ln|x+1| + \sqrt{2}\arctan\left[(x-1)/\sqrt{2}\right] + C$
23. $\ln 3$ **25.** $\frac{1}{2}\ln(8/5) - \pi/4 + \arctan 2 \approx 0.557$
27. $\ln|1 + \sec x| + C$ **29.** $\ln\left|\frac{\tan x + 2}{\tan x + 3}\right| + C$
31. $\frac{1}{5}\ln\left|\frac{e^x - 1}{e^x + 4}\right| + C$ **33.** $2\sqrt{x} + 2\ln\left|\frac{\sqrt{x} - 2}{\sqrt{x} + 2}\right| + C$
35–37. Proofs **39.** First divide x^3 by $(x-5)$.
41. (a) Substitution: $u = x^2 + 2x - 8$ (b) Partial fractions
(c) Trigonometric substitution (tan) or inverse tangent rule
43. $12\ln\left(\frac{9}{8}\right) \approx 1.4134$ **45.** 4.90 or \$490,000
47. $V = 2\pi\left(\arctan 3 - \frac{3}{10}\right) \approx 5.963$; $(\bar{x}, \bar{y}) \approx (1.521, 0.412)$
49. $x = n[e^{(n+1)kt} - 1]/[n + e^{(n+1)kt}]$ **51.** $\pi/8$

Section 8.6 *(page 555)*

1. $-\frac{1}{2}x(10-x) + 25\ln|5+x| + C$ **3.** $-\sqrt{1-x^2}/x + C$
5. $\frac{1}{24}(3x + \sin 3x \cos 3x + 2\cos^3 3x \sin 3x) + C$
7. $-2\left(\cot\sqrt{x} + \csc\sqrt{x}\right) + C$ **9.** $x - \frac{1}{2}\ln(1 + e^{2x}) + C$
11. $\frac{1}{16}x^8(8\ln x - 1) + C$
13. (a) and (b) $\frac{1}{27}e^{3x}(9x^2 - 6x + 2) + C$
15. (a) and (b) $\ln|(x+1)/x| - 1/x + C$
17. $\frac{1}{2}\left[(x^2+1)\operatorname{arccsc}(x^2+1) + \ln\left(x^2 + 1 + \sqrt{x^4 + 2x^2}\right)\right] + C$
19. $\sqrt{x^2-4}/(4x) + C$
21. $\frac{4}{25}[\ln|2-5x| + 2/(2-5x)] + C$
23. $e^x \arccos(e^x) - \sqrt{1 - e^{2x}} + C$
25. $\frac{1}{2}(x^2 + \cot x^2 + \csc x^2) + C$
27. $\left(\sqrt{2}/2\right)\arctan\left[(1 + \sin\theta)/\sqrt{2}\right] + C$
29. $-\sqrt{2 + 9x^2}/(2x) + C$
31. $\frac{1}{4}\left(2\ln|x| - 3\ln|3 + 2\ln|x||\right) + C$
33. $(3x-10)/[2(x^2 - 6x + 10)] + \frac{3}{2}\arctan(x-3) + C$
35. $\frac{1}{2}\ln\left|x^2 - 3 + \sqrt{x^4 - 6x^2 + 5}\right| + C$
37. $2/(1+e^x) - 1/[2(1+e^x)^2] + \ln(1+e^x) + C$
39. $\frac{1}{2}(e-1) \approx 0.8591$ **41.** $\frac{32}{5}\ln 2 - \frac{31}{25} \approx 3.1961$
43. $\pi/2$ **45.** $\pi^3/8 - 3\pi + 6 \approx 0.4510$ **47–51.** Proofs
53. $\frac{1}{\sqrt{5}}\ln\left|\frac{2\tan(\theta/2) - 3 - \sqrt{5}}{2\tan(\theta/2) - 3 + \sqrt{5}}\right| + C$ **55.** $\ln 2$
57. $\frac{1}{2}\ln(3 - 2\cos\theta) + C$ **59.** $-2\cos\sqrt{\theta} + C$ **61.** $4\sqrt{3}$
63. (a) $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$
$\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$
$\int x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$
65. (a) Arctangent Formula, Formula 23,
$\int \frac{1}{u^2+1}\,du,\ u = e^x$
(b) Log Rule: $\int \frac{1}{u}\,du,\ u = e^x + 1$
(c) Substitution: $u = x^2$, $du = 2x\,dx$
Then Formula 81.
(d) Integration by parts (e) Cannot be integrated
(f) Formula 16 with $u = e^{2x}$
67. False. Substitutions may first have to be made to rewrite the integral in a form that appears in the table.
69. 1919.145 ft-lb **71.** $32\pi^2$ **73.** About 401.4

Section 8.7 *(page 564)*

1.

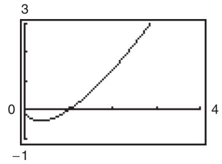
x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	1.3177	1.3332	1.3333	1.3333	1.3332	1.3177

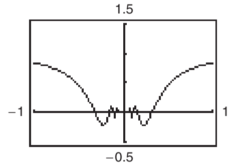
$\frac{4}{3}$

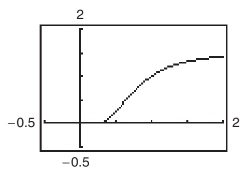
3.

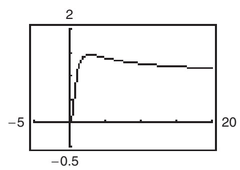
x	1	10	10^2	10^3	10^4	10^5
$f(x)$	0.9900	90,483.7	3.7×10^9	4.5×10^{10}	0	0

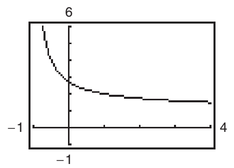
0

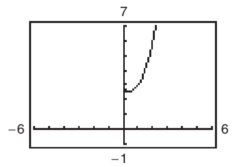
5. $\frac{3}{8}$ **7.** $\frac{1}{8}$ **9.** $\frac{5}{3}$ **11.** 4 **13.** 0 **15.** ∞ **17.** $\frac{11}{4}$
19. $\frac{3}{5}$ **21.** 1 **23.** $\frac{5}{4}$ **25.** ∞ **27.** 0 **29.** 1
31. 0 **33.** 0 **35.** ∞ **37.** $\frac{5}{9}$ **39.** 1 **41.** ∞
43. (a) Not indeterminate
(b) ∞
(c)


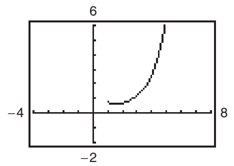
45. (a) $0 \cdot \infty$
(b) 1
(c)


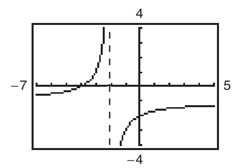
47. (a) Not indeterminate
(b) 0
(c)


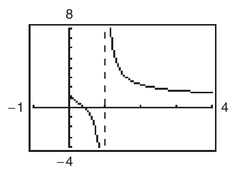
49. (a) ∞^0
(b) 1
(c)


51. (a) 1^∞ (b) e
(c)


53. (a) 0^0 (b) 3
(c)


55. (a) 0^0 (b) 1
(c)


57. (a) $\infty - \infty$ (b) $-\frac{3}{2}$
(c)


59. (a) $\infty - \infty$ (b) ∞
(c)


61. $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty - \infty$

63. Answers will vary. Examples:
(a) $f(x) = x^2 - 25, g(x) = x - 5$
(b) $f(x) = (x - 5)^2, g(x) = x^2 - 25$
(c) $f(x) = x^2 - 25, g(x) = (x - 5)^3$

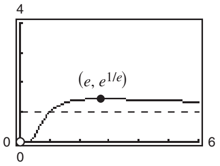
65. (a) Yes: $\frac{0}{0}$ (b) No: $\frac{0}{-1}$ (c) Yes: $\frac{\infty}{\infty}$ (d) Yes: $\frac{0}{0}$
(e) No: $\frac{-1}{0}$ (f) Yes: $\frac{0}{0}$

67.

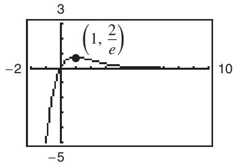
x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$	2.811	4.498	0.720	0.036	0.001	0.000

69. 0 **71.** 0 **73.** 0

75. Horizontal asymptote: $y = 1$
Relative maximum: $(e, e^{1/e})$



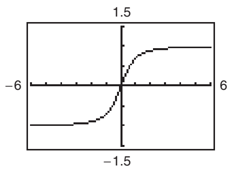
77. Horizontal asymptote: $y = 0$
Relative maximum: $(1, 2/e)$



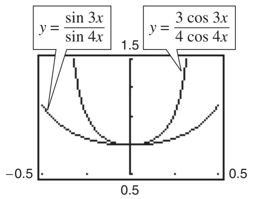
79. Limit is not of the form $0/0$ or ∞/∞.

81. Limit is not of the form $0/0$ or ∞/∞.

83. (a) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$
Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails.
(b) 1
(c)



85.



As $x \to 0$, the graphs get closer together (they both approach 0.75). By L'Hôpital's Rule,
$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3 \cos 3x}{4 \cos 4x} = \frac{3}{4}$.

87. $v = 32t + v_0$ **89.** Proof **91.** $c = \frac{2}{3}$ **93.** $c = \pi/4$

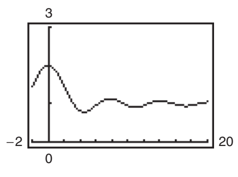
95. False. L'Hôpital's Rule does not apply because $\lim_{x\to 0}(x^2 + x + 1) \neq 0$.

97. True **99.** $\frac{3}{4}$ **101.** $\frac{4}{3}$ **103.** $a = 1, b = \pm 2$

105. Proof **107.** (a) $0 \cdot \infty$ (b) 0 **109.** Proof

111. (a)–(c) 2

113. (a)



(b) $\lim_{x\to\infty} h(x) = 1$
(c) No

115. Putnam Problem A1, 1956

Section 8.8 *(page 575)*

1. Improper; $0 \le \frac{3}{5} \le 1$

3. Not improper; continuous on $[0, 1]$

5. Not improper; continuous on $[0, 2]$

7. Improper; infinite limits of integration

9. Infinite discontinuity at $x = 0$; 4

11. Infinite discontinuity at $x = 1$; diverges

13. Infinite discontinuity at $x = 0$; diverges

15. Infinite limit of integration; converges to 1 **17.** $\frac{1}{2}$

19. Diverges **21.** Diverges **23.** 2 **25.** $1/[2(\ln 4)^2]$

27. π **29.** $\pi/4$ **31.** Diverges **33.** Diverges

35. 0 **37.** $-\frac{1}{4}$ **39.** Diverges **41.** $\pi/3$ **43.** $\ln 3$

45. $\pi/6$ **47.** $2\pi\sqrt{6}/3$ **49.** $p > 1$ **51.** Proof

53. Diverges **55.** Converges **57.** Converges

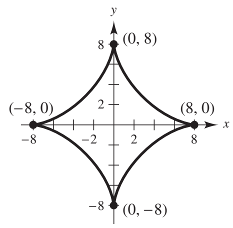
59. Diverges **61.** Converges

63. An integral with infinite integration limits, an integral with an infinite discontinuity at or between the integration limits

65. The improper integral diverges. **67.** e **69.** π

71. (a) 1 (b) $\pi/2$ (c) 2π

73.



Perimeter = 48

75. $8\pi^2$ **77.** (a) $W = 20{,}000$ mile-tons (b) 4000 mi

79. (a) Proof (b) $P = 43.53\%$ (c) $E(x) = 7$

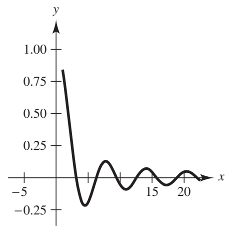
81. (a) \$757,992.41 (b) \$837,995.15 (c) \$1,066,666.67

83. $P = \left[2\pi NI\left(\sqrt{r^2 + c^2} - c\right)\right]/\left(kr\sqrt{r^2 + c^2}\right)$

85. False. Let $f(x) = 1/(x + 1)$. **87.** True

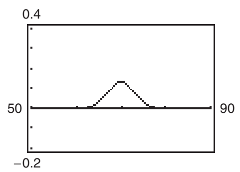
89. (a) and (b) Proofs
(c) The definition of the improper integral $\int_{-\infty}^{\infty}$ is not $\lim_{a\to\infty} \int_{-a}^{a}$ but rather that if you rewrite the integral that diverges, you can find that the integral converges.

91. (a) $\int_1^\infty \frac{1}{x^n}\,dx$ will converge if $n > 1$ and diverge if $n \le 1$.
(b)



(c) Converges

93. (a)



(b) About 0.2525
(c) 0.2525; same by symmetry

95. $1/s, s > 0$ **97.** $2/s^3, s > 0$ **99.** $s/(s^2 + a^2), s > 0$

101. $s/(s^2 - a^2), s > |a|$

103. (a) $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(3) = 2$ (b) Proof
(c) $\Gamma(n) = (n - 1)!$

105. $c = 1$; $\ln(2)$

107. $8\pi[(\ln 2)^2/3 - (\ln 4)/9 + 2/27] \approx 2.01545$

109. $\int_0^1 2 \sin(u^2)\, du$; 0.6278 **111.** Proof

Review Exercises for Chapter 8 *(page 579)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$

5. $\ln(2) + \frac{1}{2} \approx 1.1931$ **7.** $100 \arcsin(x/10) + C$

9. $\frac{1}{9}e^{3x}(3x - 1) + C$ **11.** $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + C$

13. $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$

15. $\frac{1}{16}\left[(8x^2 - 1)\arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$

17. $\sin(\pi x - 1)[\cos^2(\pi x - 1) + 2]/(3\pi) + C$

19. $\frac{2}{3}[\tan^3(x/2) + 3\tan(x/2)] + C$ **21.** $\tan\theta + \sec\theta + C$

23. $3\pi/16 + \frac{1}{2} \approx 1.0890$ **25.** $3\sqrt{4 - x^2}/x + C$

27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17} \approx 0.3675$

31. (a), (b), and (c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

33. $6\ln|x + 3| - 5\ln|x - 4| + \text{C}$

35. $\frac{1}{4}[6\ln|x - 1| - \ln(x^2 + 1) + 6\arctan x] + C$

37. $x - \frac{64}{11}\ln|x + 8| + \frac{9}{11}\ln|x - 3| + C$

39. $\frac{1}{25}[4/(4 + 5x) + \ln|4 + 5x|] + C$ **41.** $1 - \sqrt{2}/2$

43. $\frac{1}{2}\ln|x^2 + 4x + 8| - \arctan[(x + 2)/2] + C$

45. $\ln|\tan \pi x|/\pi + C$ **47.** Proof

49. $\frac{1}{8}(\sin 2\theta - 2\theta\cos 2\theta) + C$

51. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3\arctan(x^{1/4})] + C$

53. $2\sqrt{1 - \cos x} + C$ **55.** $\sin x \ln(\sin x) - \sin x + C$

57. $\frac{5}{2}\ln|(x - 5)/(x + 5)| + C$

59. $y = x\ln|x^2 + x| - 2x + \ln|x + 1| + C$ **61.** $\frac{1}{5}$

63. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **65.** π **67.** $\frac{128}{15}$

69. $(\bar{x}, \bar{y}) = (0, 4/(3\pi))$ **71.** 3.82 **73.** 0 **75.** ∞

77. 1 **79.** $1000e^{0.09} \approx 1094.17$ **81.** Converges; $\frac{32}{3}$

83. Diverges **85.** Converges; 1 **87.** Converges; $\pi/4$

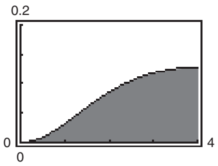
89. (a) \$6,321,205.59 (b) \$10,000,000

91. (a) 0.4581 (b) 0.0135

P.S. Problem Solving *(page 581)*

1. (a) $\frac{4}{3}, \frac{16}{15}$ (b) Proof **3.** $\ln 3$ **5.** Proof

7. (a)



Area ≈ 0.2986

(b) $\ln 3 - \frac{4}{5}$ (c) $\ln 3 - \frac{4}{5}$

9. $\ln 3 - \frac{1}{2} \approx 0.5986$

11. (a) ∞ (b) 0 (c) $-\frac{2}{3}$

The form $0 \cdot \infty$ is indeterminant.

13. About 0.8670 **15.** $\dfrac{1/12}{x} + \dfrac{1/42}{x - 3} + \dfrac{1/10}{x - 1} + \dfrac{111/140}{x + 4}$

17–19. Proofs **21.** About 0.0158

Chapter 9

Section 9.1 *(page 592)*

1. 3, 9, 27, 81, 243 **3.** 1, 0, −1, 0, 1 **5.** $2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$

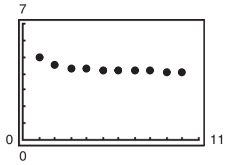
7. 3, 4, 6, 10, 18 **9.** c **10.** a **11.** d **12.** b

13. 14, 17; add 3 to preceding term.

15. 80, 160; multiply preceding term by 2. **17.** $n + 1$

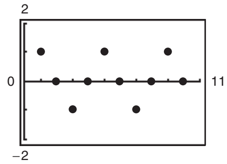
19. $1/[(2n + 1)(2n)]$ **21.** 5 **23.** 2

25.



Converges to 4

27.



Diverges

29. Converges to 0 **31.** Diverges **33.** Converges to 5

35. Converges to 0 **37.** Diverges **39.** Converges to 0

41. Converges to 1 **43.** Converges to 0

45. Answers will vary. Sample answer: $6n - 4$

47. Answers will vary. Sample answer: $n^2 - 3$

49. Answers will vary. Sample answer: $(n + 1)/(n + 2)$

51. Answers will vary. Sample answer: $(n + 1)/n$

53. Monotonic, bounded **55.** Not monotonic, bounded

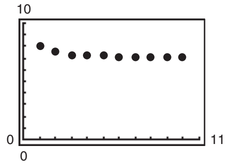
57. Monotonic, bounded **59.** Not monotonic, bounded

61. (a) $\left|7 + \frac{1}{n}\right| \ge 7 \implies$ bounded

$a_n > a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

(b)



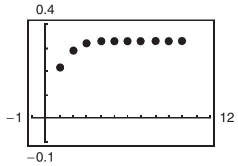
Limit = 7

63. (a) $\left|\frac{1}{3}\left(1 - \frac{1}{3^n}\right)\right| < \frac{1}{3} \implies$ bounded

$a_n < a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

(b)



Limit = $\frac{1}{3}$

65. $\{a_n\}$ has a limit because it is bounded and monotonic; because $2 \le a_n \le 4$, $2 \le L \le 4$.

67. (a) No. $\lim_{n\to\infty} A_n$ does not exist.

(b)

n	1	2	3	4
A_n	\$10,045.83	\$10,091.88	\$10,138.13	\$10,184.60

n	5	6	7
A_n	\$10,231.28	\$10,278.17	\$10,325.28

n	8	9	10
A_n	\$10,372.60	\$10,420.14	\$10,467.90

69. No. A sequence is said to converge when its terms approach a real number.

71. (a) $10 - \frac{1}{n}$

(b) Impossible. The sequence converges by Theorem 9.5.

(c) $a_n = \dfrac{3n}{4n + 1}$

(d) Impossible. An unbounded sequence diverges.

73. (a) $\$4{,}500{,}000{,}000(0.8)^n$

(b)

Year	1	2
Budget	\$3,600,000,000	\$2,880,000,000

Year	3	4
Budget	\$2,304,000,000	\$1,843,200,000

(c) Converges to 0

75. 1, 1.4142, 1.4422, 1.4142, 1.3797, 1.3480; Converges to 1

77. Proof **79.** True **81.** True

83. (a) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

(b) 1, 2, 1.5, 1.6667, 1.6, 1.6250, 1.6154, 1.6190, 1.6176, 1.6182 (c) Proof

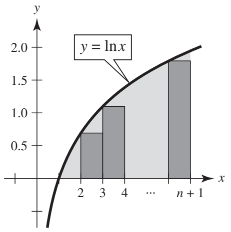
(d) $\rho = (1 + \sqrt{5})/2 \approx 1.6180$

85. (a) 1.4142, 1.8478, 1.9616, 1.9904, 1.9976

(b) $a_n = \sqrt{2 + a_{n-1}}$ (c) $\lim_{n\to\infty} a_n = 2$

87. (a) Proof

(b)



(c) Proof (d) Proof

(e) $\dfrac{\sqrt[20]{20!}}{20} \approx 0.4152$; $\dfrac{\sqrt[50]{50!}}{50} \approx 0.3897$; $\dfrac{\sqrt[100]{100!}}{100} \approx 0.3799$

89–91. Proofs **93.** Putnam Problem A1, 1990

Section 9.2 *(page 601)*

1. 1, 1.25, 1.361, 1.424, 1.464

3. 3, −1.5, 5.25, −4.875, 10.3125

5. 3, 4.5, 5.25, 5.625, 5.8125 **7.** Geometric series: $r = \frac{7}{6} > 1$

9. $\lim_{n\to\infty} a_n = 1 \neq 0$ **11.** $\lim_{n\to\infty} a_n = 1 \neq 0$

13. $\lim_{n\to\infty} a_n = \frac{1}{2} \neq 0$ **15.** Geometric series: $r = \frac{5}{6} < 1$

17. Geometric series: $r = 0.9 < 1$

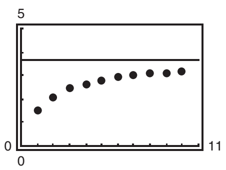
19. Telescoping series: $a_n = 1/n - 1/(n+1)$; Converges to 1.

21. (a) $\frac{11}{3}$

(b)

n	5	10	20	50	100
S_n	2.7976	3.1643	3.3936	3.5513	3.6078

(c)



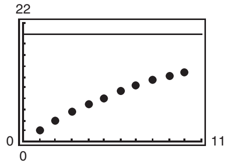
(d) The terms of the series decrease in magnitude relatively slowly, and the sequence of partial sums approaches the sum of the series relatively slowly.

23. (a) 20

(b)

n	5	10	20	50	100
S_n	8.1902	13.0264	17.5685	19.8969	19.9995

(c)



(d) The terms of the series decrease in magnitude relatively slowly, and the sequence of partial sums approaches the sum of the series relatively slowly.

25. 15 **27.** 3 **29.** 32 **31.** $\dfrac{1}{2}$ **33.** $\dfrac{\sin(1)}{1 - \sin(1)}$

35. (a) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{10}(0.1)^n$ (b) $\dfrac{4}{9}$ **37.** (a) $\displaystyle\sum_{n=0}^{\infty} \frac{81}{100}(0.01)^n$ (b) $\dfrac{9}{11}$

39. (a) $\displaystyle\sum_{n=0}^{\infty} \frac{3}{40}(0.01)^n$ (b) $\dfrac{5}{66}$ **41.** Diverges **43.** Diverges

45. Converges **47.** Diverges **49.** Diverges

51. Diverges **53.** Diverges **55.** See definitions on page 595.

57. The series given by

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots,\ a \neq 0$$

is a geometric series with ratio r. When $0 < |r| < 1$, the series converges to the sum $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

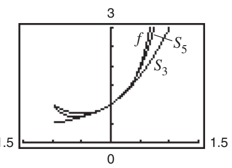
59. The series in (a) and (b) are the same. The series in (c) is different unless $a_1 = a_2 = \cdots = a$ is constant.

61. $|x| < \dfrac{1}{3}$; $\dfrac{3x}{1-3x}$ **63.** $0 < x < 2$; $(x-1)/(2-x)$

65. $-1 < x < 1$; $1/(1+x)$

67. (a) x (b) $f(x) = 1/(1-x)$, $|x| < 1$

(c)



Answers will vary.

69. The required terms for the two series are $n = 100$ and $n = 5$, respectively. The second series converges at a higher rate.

71. $160{,}000(1 - 0.95^n)$ units

73. $\displaystyle\sum_{i=0}^{\infty} 200(0.75)^i$; Sum = \$800 million **75.** 152.42 feet

77. $\dfrac{1}{8}$; $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^n = \frac{1/2}{1 - 1/2} = 1$

79. (a) $\displaystyle -1 + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = -1 + \frac{a}{1-r} = -1 + \frac{1}{1 - 1/2} = 1$

(b) No (c) 2

81. (a) 126 in.^{2} (b) 128 in.^{2}

83. The \$2,000,000 sweepstakes has a present value of \$1,146,992.12. After accruing interest over the 20-year period, it attains its full value.

85. (a) \$5,368,709.11 (b) \$10,737,418.23 (c) \$21,474,836.47

87. (a) \$14,773.59 (b) \$14,779.65

89. (a) \$91,373.09 (b) \$91,503.32

91. False. $\displaystyle\lim_{n\to\infty} \frac{1}{n} = 0$, but $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

93. False. $\displaystyle\sum_{n=1}^{\infty} ar^n = \left(\frac{a}{1-r}\right) - a$; The formula requires that the geometric series begins with $n = 0$.

95. True **97.** Answers will vary. Example: $\displaystyle\sum_{n=0}^{\infty} 1,\ \sum_{n=0}^{\infty} (-1)$

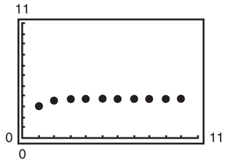
99–101. Proofs **103.** 2

Section 9.3 *(page 609)*

1. Diverges **3.** Converges **5.** Converges
7. Converges **9.** Diverges **11.** Diverges
13. Converges **15.** Converges **17.** Converges
19. Diverges **21.** Converges **23.** Diverges
25. $f(x)$ is not positive for $x \geq 1$.
27. $f(x)$ is not always decreasing. **29.** Converges
31. Diverges **33.** Diverges **35.** Converges
37. Converges

39. (a)

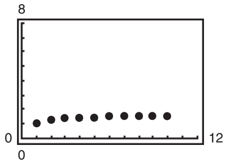
n	5	10	20	50	100
S_n	3.7488	3.75	3.75	3.75	3.75



The partial sums approach the sum 3.75 very quickly.

(b)

n	5	10	20	50	100
S_n	1.4636	1.5498	1.5962	1.6251	1.635

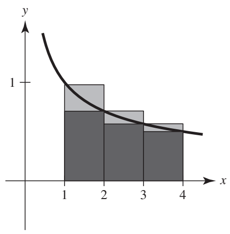


The partial sums approach the sum $\pi^2/6 \approx 1.6449$ more slowly than the series in part (a).

41. See Theorem 9.10 on page 605. Answers will vary. For example, convergence or divergence can be determined for the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

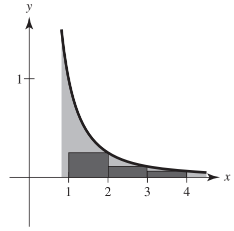
43. No. Because $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=10{,}000}^{\infty} \frac{1}{n}$ also diverges. The convergence or divergence of a series is not determined by the first finite number of terms of the series.

45. (a)



The area under the rectangles is greater than the area under the curve. Because $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^{\infty} = \infty$ diverges, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.

(b)



The area under the rectangles is less than the area under the curve. Because $\int_1^{\infty} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^{\infty} = 1$ converges, $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges $\left(\text{and so does } \sum_{n=1}^{\infty} \frac{1}{n^2}\right)$.

47. $p > 1$ **49.** $p > 1$ **51.** $p > 3$ **53.** Proof

55. $S_5 = 1.4636$, $R_5 = 0.20$ **57.** $S_{10} \approx 0.9818$, $R_{10} \approx 0.0997$ **59.** $S_4 \approx 0.4049$, $R_4 \approx 5.6 \times 10^{-8}$

61. $N \geq 7$ **63.** $N \geq 16$

65. (a) $\sum_{n=2}^{\infty} \frac{1}{n^{1.1}}$ converges by the p-Series Test because $1.1 > 1$. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges by the Integral Test because $\int_2^{\infty} \frac{1}{x \ln x}\,dx$ diverges.

(b) $$\sum_{n=2}^{\infty} \frac{1}{n^{1.1}} = 0.4665 + 0.2987 + 0.2176 + 0.1703 + 0.1393 + \cdots$$
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} = 0.7213 + 0.3034 + 0.1803 + 0.1243 + 0.0930 + \cdots$$

(c) $n \geq 3.431 \times 10^{15}$

67. (a) Let $f(x) = 1/x$. f is positive, continuous, and decreasing on $[1, \infty)$.
$$S_n - 1 \leq \int_1^n \frac{1}{x}\,dx = \ln n$$
$$S_n \geq \int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$$
So, $\ln(n+1) \leq S_n \leq 1 + \ln n$.

(b) $\ln(n+1) - \ln n \leq S_n - \ln n \leq 1$
Also, $\ln(n+1) - \ln n > 0$ for $n \geq 1$. So, $0 \leq S_n - \ln n \leq 1$, and the sequence $\{a_n\}$ is bounded.

(c) $$a_n - a_{n+1} = [S_n - \ln n] - [S_{n+1} - \ln(n+1)] = \int_n^{n+1} \frac{1}{x}\,dx - \frac{1}{n+1} \geq 0$$
So, $a_n \geq a_{n+1}$.

(d) Because the sequence is bounded and monotonic, it converges to a limit, γ.

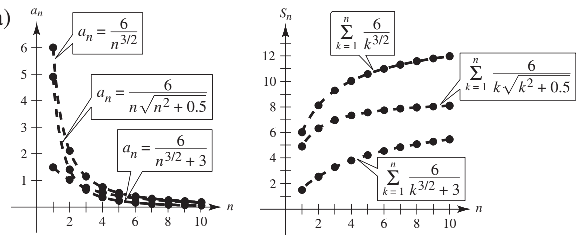
(e) 0.5822

69. (a) Diverges (b) Diverges
(c) $\sum_{n=2}^{\infty} x^{\ln n}$ converges for $x < 1/e$.

71. Diverges **73.** Converges **75.** Converges
77. Diverges **79.** Diverges **81.** Converges

Section 9.4 (page 616)

1. (a)



(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$; Converges

(c) The magnitudes of the terms are less than the magnitudes of the terms of the p-series. Therefore, the series converges.

(d) The smaller the magnitudes of the terms, the smaller the magnitudes of the terms of the sequence of partial sums.

3. Diverges **5.** Diverges **7.** Diverges **9.** Converges

11. Converges **13.** Diverges **15.** Diverges

17. Converges **19.** Converges **21.** Diverges

23. Diverges; p-Series Test

25. Converges; Direct Comparison Test with $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n$

27. Diverges; nth-Term Test **29.** Converges; Integral Test

31. $\displaystyle\lim_{n\to\infty} \frac{a_n}{1/n} = \lim_{n\to\infty} na_n$; $\displaystyle\lim_{n\to\infty} na_n \neq 0$, but is finite.

The series diverges by the Limit Comparison Test.

33. Diverges **35.** Converges

37. $\displaystyle\lim_{n\to\infty} n\left(\frac{n^3}{5n^4+3}\right) = \frac{1}{5} \neq 0$; So, $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5n^4+3}$ diverges.

39. Diverges **41.** Converges

43. Convergence or divergence is dependent on the form of the general term for the series and not necessarily on the magnitudes of the terms.

45. See Theorem 9.13 on page 614. Answers will vary. For example, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n-1}}$ diverges because $\displaystyle\lim_{n\to\infty} \frac{1/\sqrt{n-1}}{1/\sqrt{n}} = 1$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges (p-series).

47. (a) Proof

(b)

n	5	10	20	50	100
S_n	1.1839	1.2087	1.2212	1.2287	1.2312

(c) 0.1226 (d) 0.0277

49. False. Let $a_n = 1/n^3$ and $b_n = 1/n^2$. **51.** True

53. True **55.** Proof **57.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n^3}$ **59–65.** Proofs

67. Putnam Problem B4, 1988

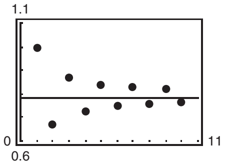
Section 9.5 (page 625)

1. (a)

n	1	2	3	4	5
S_n	1.0000	0.6667	0.8667	0.7238	0.8349

n	6	7	8	9	10
S_n	0.7440	0.8209	0.7543	0.8131	0.7605

(b)



(c) The points alternate sides of the horizontal line $y = \pi/4$ that represents the sum of the series. The distances between the successive points and the line decrease.

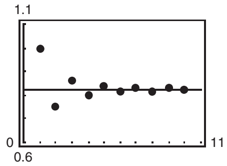
(d) The distance in part (c) is always less than the magnitude of the next term of the series.

3. (a)

n	1	2	3	4	5
S_n	1.0000	0.7500	0.8611	0.7986	0.8386

n	6	7	8	9	10
S_n	0.8108	0.8312	0.8156	0.8280	0.8180

(b)



(c) The points alternate sides of the horizontal line $y = \pi^2/12$ that represents the sum of the series. The distances between the successive points and the line decrease.

(d) The distance in part (c) is always less than the magnitude of the next term of the series.

5. Converges **7.** Converges **9.** Diverges **11.** Diverges

13. Converges **15.** Diverges **17.** Diverges

19. Converges **21.** Converges **23.** Converges

25. Converges **27.** $1.8264 \leq S \leq 1.8403$

29. $1.7938 \leq S \leq 1.8054$ **31.** 10 **33.** 7

35. 7 terms (Note that the sum begins with $n = 0$.)

37. Converges absolutely **39.** Converges absolutely

41. Converges conditionally **43.** Diverges

45. Converges conditionally **47.** Converges absolutely

49. Converges absolutely **51.** Converges conditionally

53. Converges absolutely

55. An alternating series is a series whose terms alternate in sign.

57. $|S - S_N| = |R_N| \leq a_{N+1}$

59. (a) False. For example, let $a_n = \dfrac{(-1)^n}{n}$.

Then $\displaystyle\sum a_n = \sum \frac{(-1)^n}{n}$ converges

and $\displaystyle\sum(-a_n) = \sum \frac{(-1)^{n+1}}{n}$ converges.

But, $\displaystyle\sum |a_n| = \sum \frac{1}{n}$ diverges.

(b) True. For if $\sum |a_n|$ converged, then so would $\sum a_n$ by Theorem 9.16.

61. True **63.** $p > 0$

65. Proof; The converse is false. For example: Let $a_n = 1/n$.

67. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, hence so does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.

69. (a) No. $a_{n+1} \leq a_n$ is not satisfied for all n. For example, $\frac{1}{9} < \frac{1}{8}$.

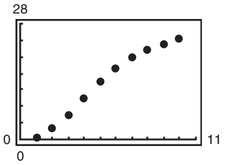
(b) Yes. 0.5

71. Converges; p-Series Test **73.** Diverges; nth-Term Test
75. Converges; Geometric Series Test
77. Converges; Integral Test
79. Converges; Alternating Series Test
81. The first term of the series is 0, not 1. You cannot regroup series terms arbitrarily.

Section 9.6 *(page 633)*

1–3. Proofs **5.** d **6.** c **7.** f **8.** b **9.** a
10. e
11. (a) Proof
(b)

n	5	10	15	20	25
S_n	13.7813	24.2363	25.8468	25.9897	25.9994

(c)


(d) 26

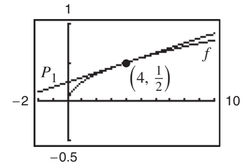
(e) The more rapidly the terms of the series approach 0, the more rapidly the sequence of partial sums approaches the sum of the series.

13. Converges **15.** Diverges **17.** Diverges
19. Converges **21.** Converges **23.** Converges
25. Diverges **27.** Converges **29.** Converges
31. Diverges **33.** Converges **35.** Converges
37. Converges **39.** Diverges **41.** Converges
43. Diverges **45.** Converges **47.** Converges
49. Converges **51.** Converges; Alternating Series Test
53. Converges; p-Series Test **55.** Diverges; nth-Term Test
57. Diverges; Geometric Series Test
59. Converges; Limit Comparison Test with $b_n = 1/2^n$
61. Converges; Direct Comparison Test with $b_n = 1/3^n$
63. Diverges; Ratio Test **65.** Converges; Ratio Test
67. Converges; Ratio Test **69.** a and c **71.** a and b
73. $\sum_{n=0}^{\infty} \frac{n+1}{7^{n+1}}$ **75.** (a) 9 (b) -0.7769
77. Diverges; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$
79. Converges; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$ **81.** Diverges; $\lim a_n \neq 0$
83. Converges **85.** Converges **87.** $(-3, 3)$
89. $(-2, 0]$ **91.** $x = 0$
93. See Theorem 9.17 on page 627.
95. No; the series $\sum_{n=1}^{\infty} \frac{1}{n + 10{,}000}$ diverges.
97. Absolutely; by Theorem 9.17 **99–105.** Proofs
107. (a) Diverges (b) Converges (c) Converges
(d) Converges for all integers $x \geq 2$
109. Putnam Problem 7, morning session, 1951

Section 9.7 *(page 658)*

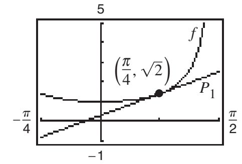
1. d **2.** c **3.** a **4.** b

5. $P_1 = \frac{1}{16}x + \frac{1}{4}$



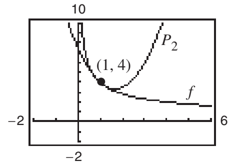
P_1 is the first-degree Taylor polynomial for f at 4.

7. $P_1 = \sqrt{2}x + \sqrt{2}(4 - \pi)/4$



P_1 is the first-degree Taylor polynomial for f at $\pi/4$.

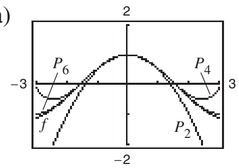
9.



x	0	0.8	0.9	1	1.1
$f(x)$	Error	4.4721	4.2164	4.0000	3.8139
$P_2(x)$	7.5000	4.4600	4.2150	4.0000	3.8150

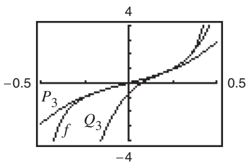
x	1.2	2
$f(x)$	3.6515	2.8284
$P_2(x)$	3.6600	3.5000

11. (a)



(b) $f^{(2)}(0) = -1$ $P_2^{(2)}(0) = -1$
$f^{(4)}(0) = 1$ $P_4^{(4)}(0) = 1$
$f^{(6)}(0) = -1$ $P_6^{(6)}(0) = -1$
(c) $f^{(n)}(0) = P_n^{(n)}(0)$

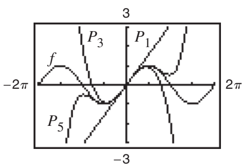
13. $1 + 4x + 8x^2 + \frac{32}{3}x^3 + \frac{32}{3}x^4$
15. $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3 + \frac{1}{384}x^4$ **17.** $x - \frac{1}{6}x^3 + \frac{1}{120}x^5$
19. $x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$ **21.** $1 - x + x^2 - x^3 + x^4 - x^5$
23. $1 + \frac{1}{2}x^2$ **25.** $2 - 2(x - 1) + 2(x - 1)^2 - 2(x - 1)^3$
27. $2 + \frac{1}{4}(x - 4) - \frac{1}{64}(x - 4)^2 + \frac{1}{512}(x - 4)^3$
29. $\ln 2 + \frac{1}{2}(x - 2) - \frac{1}{8}(x - 2)^2 + \frac{1}{24}(x - 2)^3 - \frac{1}{64}(x - 2)^4$
31. (a) $P_3(x) = \pi x + \frac{\pi^3}{3}x^3$
(b) $Q_3(x) = 1 + 2\pi\left(x - \frac{1}{4}\right) + 2\pi^2\left(x - \frac{1}{4}\right)^2 + \frac{8\pi^3}{3}\left(x - \frac{1}{4}\right)^3$



33. (a)

x	0	0.25	0.50	0.75	1.00
$\sin x$	0	0.2474	0.4794	0.6816	0.8415
$P_1(x)$	0	0.25	0.50	0.75	1.00
$P_3(x)$	0	0.2474	0.4792	0.6797	0.8333
$P_5(x)$	0	0.2474	0.4794	0.6817	0.8417

(b)



(c) As the distance increases, the polynomial approximation becomes less accurate.

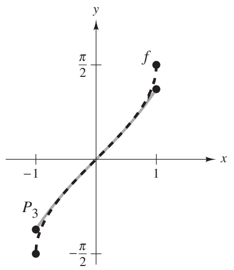
35. (a) $P_3(x) = x + \frac{1}{6}x^3$

(b)

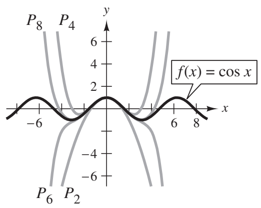
x	-0.75	-0.50	-0.25	0	0.25
$f(x)$	-0.848	-0.524	-0.253	0	0.253
$P_3(x)$	-0.820	-0.521	-0.253	0	0.253

x	0.50	0.75
$f(x)$	0.524	0.848
$P_3(x)$	0.521	0.820

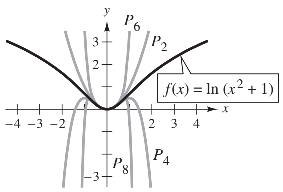
(c)



37.



39.



41. 2.7083 **43.** 0.7419 **45.** $R_4 \le 2.03 \times 10^{-5}$; 0.000001

47. $R_3 \le 7.82 \times 10^{-3}$; 0.00085 **49.** 3 **51.** 5

53. $n = 9$; $\ln(1.5) \approx 0.4055$ **55.** $-0.3936 < x < 0$

57. $-0.9467 < x < 0.9467$

59. The graphs of the approximating polynomial P and the elementary function f both pass through the point $(c, f(c))$, and the slope of the graph of P is the same as the slope of the graph of f at the point $(c, f(c))$. If P is of degree n, then the first n derivatives of f and P agree at c. This allows the graph of P to resemble the graph of f near the point $(c, f(c))$.

61. See "Definitions of nth Taylor Polynomial and nth Maclaurin Polynomial" on page 638.

63. As the degree of the polynomial increases, the graph of the Taylor polynomial becomes a better and better approximation of the function within the interval of convergence. Therefore, the accuracy is increased.

65. (a) $f(x) \approx P_4(x) = 1 + x + (1/2)x^2 + (1/6)x^3 + (1/24)x^4$

$g(x) \approx Q_5(x) = x + x^2 + (1/2)x^3 + (1/6)x^4 + (1/24)x^5$

$Q_5(x) = xP_4(x)$

(b) $g(x) \approx P_6(x) = x^2 - x^4/3! + x^6/5!$

(c) $g(x) \approx P_4(x) = 1 - x^2/3! + x^4/5!$

67. (a) $Q_2(x) = -1 + (\pi^2/32)(x + 2)^2$

(b) $R_2(x) = -1 + (\pi^2/32)(x - 6)^2$

(c) No. Horizontal translations of the result in part (a) are possible only at $x = -2 + 8n$ (where n is an integer) because the period of f is 8.

69. Proof

71. As you move away from $x = c$, the Taylor polynomial becomes less and less accurate.

Section 9.8 *(page 654)*

1. 0 **3.** 2 **5.** $R = 1$ **7.** $R = \frac{1}{4}$ **9.** $R = \infty$

11. $(-4, 4)$ **13.** $(-1, 1]$ **15.** $(-\infty, \infty)$ **17.** $x = 0$

19. $(-6, 6)$ **21.** $(-5, 13]$ **23.** $(0, 2]$ **25.** $(0, 6)$

27. $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **29.** $(-\infty, \infty)$ **31.** $(-1, 1)$ **33.** $x = 3$

35. $R = c$ **37.** $(-k, k)$ **39.** $(-1, 1)$

41. $\displaystyle\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$ **43.** $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$

45. (a) $(-3, 3)$ (b) $(-3, 3)$ (c) $(-3, 3)$ (d) $[-3, 3)$

47. (a) $(0, 2]$ (b) $(0, 2)$ (c) $(0, 2)$ (d) $[0, 2]$

49. A series of the form

$$\sum_{n=0}^{\infty} a_n(x - c)^n = a_0 + a_1(x - c) + a_2(x - c)^2 + \cdots + a_n(x - c)^n + \cdots$$

is called a power series centered at c, where c is a constant.

51. The interval of convergence of a power series is the set of all values of x for which the power series converges.

53. You differentiate and integrate the power series term by term. The radius of convergence remains the same. However, the interval of convergence might change.

55. Many answers possible.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n$ Geometric: $\left|\frac{x}{2}\right| < 1 \implies |x| < 2$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ converges for $-1 < x \le 1$

(c) $\displaystyle\sum_{n=1}^{\infty} (2x + 1)^n$ Geometric:

$|2x + 1| < 1 \implies -1 < x < 0$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x - 2)^n}{n4^n}$ converges for $-2 \le x < 6$

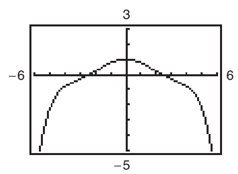
57. (a) For $f(x)$: $(-\infty, \infty)$; For $g(x)$: $(-\infty, \infty)$

(b) Proof (c) Proof (d) $f(x) = \sin x$; $g(x) = \cos x$

59–63. Proofs

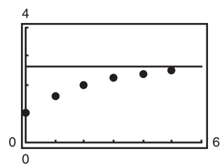
65. (a) Proof (b) Proof

(c)

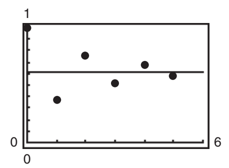


(d) 0.92

67. (a) $\frac{8}{3}$



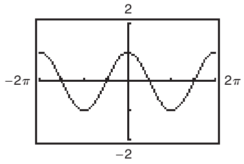
(b) $\frac{8}{13}$



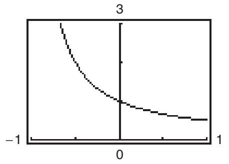
(c) The alternating series converges more rapidly. The partial sums of the series of positive terms approach the sum from below. The partial sums of the alternating series alternate sides of the horizontal line representing the sum.

(d)

M	10	100	1000	10,000
N	5	14	24	35

69. 

$f(x) = \cos x$

71. 

$f(x) = 1/(1 + x)$

73. False. Let $a_n = (-1)^n/(n2^n)$. **75.** True **77.** Proof

79. (a) $(-1, 1)$ (b) $f(x) = (c_0 + c_1x + c_2x^2)/(1 - x^3)$

81. Proof

Section 9.9 *(page 662)*

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{4^{n+1}}$ **3.** $\displaystyle\sum_{n=0}^{\infty} \frac{4}{3}\left(\frac{-x}{3}\right)^n$

5. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^{n+1}}$ **7.** $\displaystyle\sum_{n=0}^{\infty} (3x)^n$ **9.** $\displaystyle -\frac{5}{9}\sum_{n=0}^{\infty}\left[\frac{2}{9}(x+3)\right]^n$

$(-1, 3)$ $\left(-\frac{1}{3}, \frac{1}{3}\right)$ $\left(-\frac{15}{2}, \frac{3}{2}\right)$

11. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1} x^n}{4^{n+1}}$ **13.** $\displaystyle\sum_{n=0}^{\infty}\left[\frac{1}{(-3)^n} - 1\right]x^n$

$\left(-\frac{4}{3}, \frac{4}{3}\right)$ $(-1, 1)$

15. $\displaystyle\sum_{n=0}^{\infty} x^n[1 + (-1)^n] = 2\sum_{n=0}^{\infty} x^{2n}$ **17.** $\displaystyle 2\sum_{n=0}^{\infty} x^{2n}$

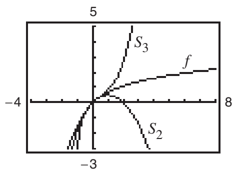
$(-1, 1)$ $(-1, 1)$

19. $\displaystyle\sum_{n=1}^{\infty} n(-1)^n x^{n-1}$ **21.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

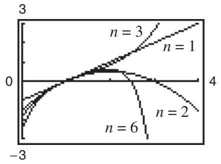
$(-1, 1)$ $(-1, 1]$

23. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$ **25.** $\displaystyle\sum_{n=0}^{\infty} (-1)^n (2x)^{2n}$

$(-1, 1)$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$

27. 

x	0.0	0.2	0.4	0.6	0.8	1.0
S_2	0.000	0.180	0.320	0.420	0.480	0.500
$\ln(x + 1)$	0.000	0.182	0.336	0.470	0.588	0.693
S_3	0.000	0.183	0.341	0.492	0.651	0.833

29. (a) 

(b) $\ln x, 0 < x \le 2, R = 1$
(c) -0.6931
(d) $\ln(0.5)$; The error is approximately 0.

31. 0.245 **33.** 0.125 **35.** $\displaystyle\sum_{n=1}^{\infty} nx^{n-1},\ -1 < x < 1$

37. $\displaystyle\sum_{n=0}^{\infty} (2n+1)x^n,\ -1 < x < 1$

39. $E(n) = 2$. Because the probability of obtaining a head on a single toss is $\frac{1}{2}$, it is expected that, on average, a head will be obtained in two tosses.

41. Because $\displaystyle\frac{1}{1+x} = \frac{1}{1-(-x)}$, substitute $(-x)$ into the geometric series.

43. Because $\displaystyle\frac{5}{1+x} = 5\left(\frac{1}{1-(-x)}\right)$, substitute $(-x)$ into the geometric series and then multiply the series by 5.

45. Proof **47.** (a) Proof (b) 3.14

49. $\ln\frac{3}{2} \approx 0.4055$; See Exercise 21.

51. $\ln\frac{7}{5} \approx 0.3365$; See Exercise 49.

53. $\arctan\frac{1}{2} \approx 0.4636$; See Exercise 52.

55. The series in Exercise 52 converges to its sum at a lower rate because its terms approach 0 at a much lower rate.

57. The series converges on the interval $(-5, 3)$ and perhaps also at one or both endpoints.

59. $\sqrt{3}\pi/6$ **61.** $S_1 = 0.3183098862,\ 1/\pi \approx 0.3183098862$

Section 9.10 *(page 673)*

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ **3.** $\displaystyle\frac{\sqrt{2}}{2}\sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}}{n!}\left(x - \frac{\pi}{4}\right)^n$

5. $\displaystyle\sum_{n=0}^{\infty} (-1)^n(x-1)^n$ **7.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^{n+1}}{n+1}$

9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(3x)^{2n+1}}{(2n+1)!}$ **11.** $1 + x^2/2! + 5x^4/4! + \cdots$

13–15. Proofs **17.** $\displaystyle\sum_{n=0}^{\infty} (-1)^n(n+1)x^n$

19. $\displaystyle 1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)x^n}{2^n n!}$

21. $\displaystyle \frac{1}{2}\left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)x^{2n}}{2^{3n} n!}\right]$

23. $\displaystyle 1 + \frac{x}{2} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdots (2n-3)x^n}{2^n n!}$

25. $\displaystyle 1 + \frac{x^2}{2} + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdots (2n-3)x^{2n}}{2^n n!}$

27. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$ **29.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$ **31.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(3x)^{2n+1}}{(2n+1)!}$

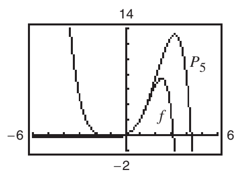
33. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$ **35.** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}$

37. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ **39.** $\displaystyle\frac{1}{2}\left[1 + \sum_{n=0}^{\infty} \frac{(-1)^n(2x)^{2n}}{(2n)!}\right]$

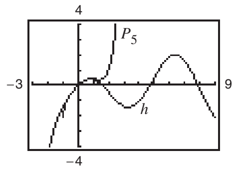
41. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!}$ **43.** $\begin{cases} \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}, & x \ne 0 \\ 1, & x = 0 \end{cases}$

45. Proof

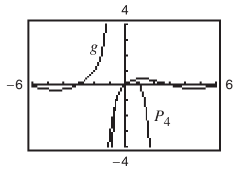
47. $P_5(x) = x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$



49. $P_5(x) = x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{3}{40}x^5$



51. $P_4(x) = x - x^2 + \frac{5}{6}x^3 - \frac{5}{6}x^4$

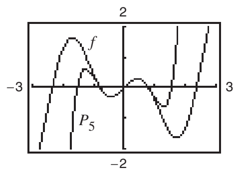


53. $\sum_{n=0}^{\infty} \frac{(-1)^{(n+1)}x^{2n+3}}{(2n+3)(n+1)!}$ **55.** 0.6931 **57.** 7.3891

59. 0 **61.** 1 **63.** 0.8075 **65.** 0.9461 **67.** 0.4872

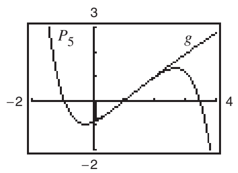
69. 0.2010 **71.** 0.7040 **73.** 0.3412

75. $P_5(x) = x - 2x^3 + \frac{2}{3}x^5$



$\left[-\frac{3}{4}, \frac{3}{4}\right]$

77. $P_5(x) = (x - 1) - \frac{1}{24}(x - 1)^3 + \frac{1}{24}(x - 1)^4 - \frac{71}{1920}(x - 1)^5$



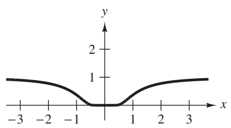
$\left[\frac{1}{4}, 2\right]$

79. See "Guidelines for Finding a Taylor Series" on page 668.

81. (a) Replace x with $(-x)$. (b) Replace x with $3x$.
(c) Multiply series by x.

83. Proof

85. (a)



(b) Proof
(c) $\sum_{n=0}^{\infty} 0x^n = 0 \neq f(x)$

87. Proof **89.** 10 **91.** -0.0390625 **93.** $\sum_{n=0}^{\infty} \binom{k}{n} x^n$

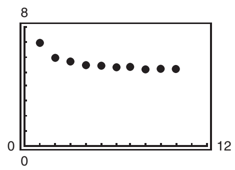
95. Proof

Review Exercises for Chapter 9 *(page 676)*

1. 5, 25, 125, 625, 3125 **3.** $-\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \frac{1}{256}, -\frac{1}{1024}$ **5.** a

6. c **7.** d **8.** b

9.



Converges to 5

11. Converges to 5 **13.** Diverges **15.** Converges to 0

17. Converges to 0 **19.** $a_n = 5n - 2$ **21.** $a_n = \frac{1}{(n! + 1)}$

23. (a)

n	1	2	3	4
A_n	\$8100.00	\$8201.25	\$8303.77	\$8407.56

n	5	6	7	8
A_n	\$8512.66	\$8619.07	\$8726.80	\$8835.89

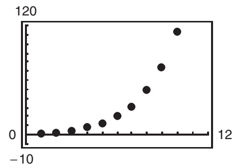
(b) \$13,148.96

25. 3, 4.5, 5.5, 6.25, 6.85

27. (a)

n	5	10	15	20	25
S_n	13.2	113.3	873.8	6648.5	50,500.3

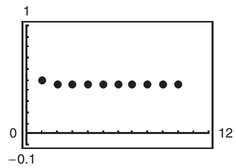
(b)



29. (a)

n	5	10	15	20	25
S_n	0.4597	0.4597	0.4597	0.4597	0.4597

(b)



31. $\frac{5}{3}$ **33.** 5.5 **35.** (a) $\sum_{n=0}^{\infty} (0.09)(0.01)^n$ (b) $\frac{1}{11}$

37. Diverges **39.** Diverges **41.** $45\frac{1}{3}$ m **43.** Diverges

45. Converges **47.** Diverges **49.** Diverges

51. Converges **53.** Diverges **55.** Converges

57. Converges **59.** Diverges **61.** Diverges

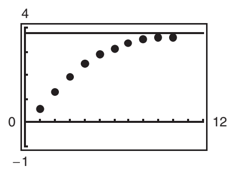
63. Converges **65.** Diverges

67. (a) Proof

(b)

n	5	10	15	20	25
S_n	2.8752	3.6366	3.7377	3.7488	3.7499

(c)



(d) 3.75

69. $P_3(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3$

71. $P_3(x) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3$ **73.** 3 terms

75. $(-10, 10)$ **77.** $[1, 3]$ **79.** Converges only at $x = 2$

81. (a) $(-5, 5)$ (b) $(-5, 5)$ (c) $(-5, 5)$ (d) $[-5, 5)$

83. Proof **85.** $\sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{x}{3}\right)^n$ **87.** $\sum_{n=0}^{\infty} 2\left(\frac{x-1}{3}\right)^n$; $(-2, 4)$

89. $\ln \frac{5}{4} \approx 0.2231$ **91.** $e^{1/2} \approx 1.6487$

93. $\cos \frac{2}{3} \approx 0.7859$ **95.** $\frac{\sqrt{2}}{2}\sum_{n=0}^{\infty} \frac{(-1)^{n(n+1)/2}}{n!}\left(x - \frac{3\pi}{4}\right)^n$

97. $\sum_{n=0}^{\infty} \frac{(x \ln 3)^n}{n!}$ **99.** $-\sum_{n=0}^{\infty} (x + 1)^n$

101. $1 + x/5 - 2x^2/25 + 6x^3/125 - 21x^4/625 + \cdots$

103. (a)–(c) $1 + 2x + 2x^2 + \frac{4}{3}x^3$ **105.** $\sum_{n=0}^{\infty} \frac{(6x)^n}{n!}$

107. $\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$ **109.** 0

P.S. Problem Solving *(page 679)*

1. (a) 1 (b) Answers will vary. Example: $0, \frac{1}{3}, \frac{2}{3}$ (c) 0

3. Proof **5.** (a) Proof (b) Yes (c) Any distance

7. (a) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{(n+2)n!}; \frac{1}{2}$ (b) $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n!}$; 5.4366

9. For $a = b$, the series converges conditionally. For no values of a and b does the series converge absolutely.

11. Proof **13.** (a) Proof (b) Proof

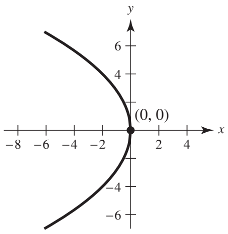
15. (a) The height is infinite. (b) The surface area is infinite. (c) Proof

Chapter 10

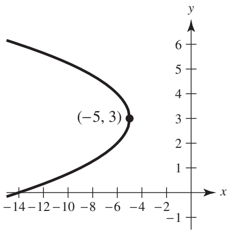
Section 10.1 *(page 692)*

1. a **2.** e **3.** c **4.** b **5.** f **6.** d

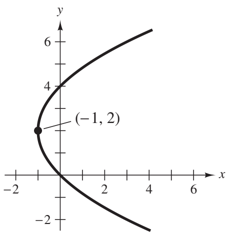
7. Vertex: $(0, 0)$
Focus: $(-2, 0)$
Directrix: $x = 2$



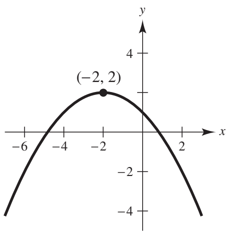
9. Vertex: $(-5, 3)$
Focus: $\left(-\frac{21}{4}, 3\right)$
Directrix: $x = -\frac{19}{4}$



11. Vertex: $(-1, 2)$
Focus: $(0, 2)$
Directrix: $x = -2$



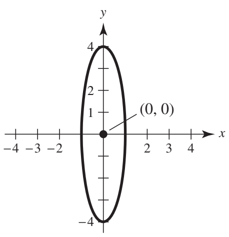
13. Vertex: $(-2, 2)$
Focus: $(-2, 1)$
Directrix: $y = 3$



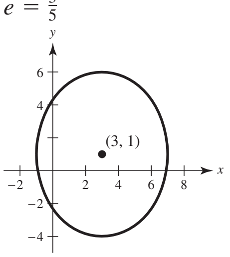
15. $y^2 - 8y + 8x - 24 = 0$ **17.** $x^2 - 32y + 160 = 0$

19. $x^2 + y - 4 = 0$ **21.** $5x^2 - 14x - 3y + 9 = 0$

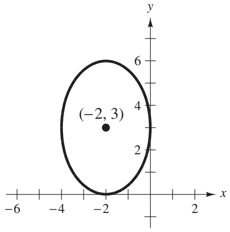
23. Center: $(0, 0)$
Foci: $\left(0, \pm\sqrt{15}\right)$
Vertices: $(0, \pm 4)$
$e = \sqrt{15}/4$



25. Center: $(3, 1)$
Foci: $(3, 4), (3, -2)$
Vertices: $(3, 6), (3, -4)$
$e = \frac{3}{5}$



27. Center: $(-2, 3)$
Foci: $\left(-2, 3 \pm \sqrt{5}\right)$
Vertices: $(-2, 6), (-2, 0)$
$e = \sqrt{5}/3$

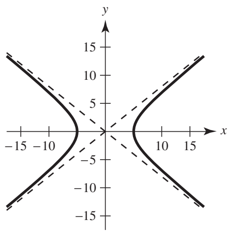


29. $x^2/36 + y^2/11 = 1$

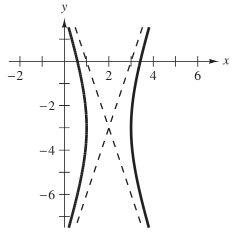
31. $(x - 3)^2/9 + (y - 5)^2/16 = 1$

33. $x^2/16 + 7y^2/16 = 1$

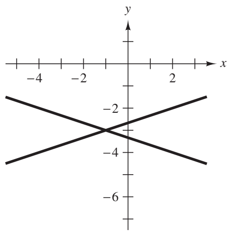
35. Center: $(0, 0)$
Vertices: $(\pm 5, 0)$
Foci: $(\pm\sqrt{41}, 0)$
Asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{4}{5}x$



37. Center: $(2, -3)$
Foci: $\left(2 \pm \sqrt{10}, -3\right)$
Vertices: $(1, -3), (3, -3)$



39. Degenerate hyperbola
Graph is two lines: $y = -3 \pm \frac{1}{3}(x + 1)$, intersecting at $(-1, -3)$.



41. $x^2/1 - y^2/25 = 1$ **43.** $y^2/9 - (x - 2)^2/(9/4) = 1$

45. $y^2/4 - x^2/12 = 1$ **47.** $(x - 3)^2/9 - (y - 2)^2/4 = 1$

49. (a) $\left(6, \sqrt{3}\right)$: $2x - 3\sqrt{3}y - 3 = 0$
$\left(6, -\sqrt{3}\right)$: $2x + 3\sqrt{3}y - 3 = 0$
(b) $\left(6, \sqrt{3}\right)$: $9x + 2\sqrt{3}y - 60 = 0$
$\left(6, -\sqrt{3}\right)$: $9x - 2\sqrt{3}y - 60 = 0$

51. Ellipse **53.** Parabola **55.** Circle **57.** Hyperbola

59. (a) A parabola is the set of all points (x, y) that are equidistant from a fixed line and a fixed point not on the line.
(b) For directrix $y = k - p$: $(x - h)^2 = 4p(y - k)$
For directrix $x = h - p$: $(y - k)^2 = 4p(x - h)$
(c) If P is a point on a parabola, then the tangent line to the parabola at P makes equal angles with the line passing through P and the focus, and with the line passing through P parallel to the axis of the parabola.

61. (a) A hyperbola is the set of all points (x, y) for which the absolute value of the difference between the distances from two distinct fixed points is constant.

(b) Transverse axis is horizontal: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

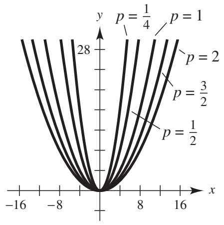
Transverse axis is vertical: $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

(c) Transverse axis is horizontal:
$y = k + (b/a)(x - h)$ and $y = k - (b/a)(x - h)$
Transverse axis is vertical:
$y = k + (a/b)(x - h)$ and $y = k - (a/b)(x - h)$

63. (a) Ellipse (b) Hyperbola (c) Circle
(d) Sample answer: Eliminate the y^2-term.

65. $\frac{9}{4}$ m **67.** (a) Proof (b) Point of intersection: $(3, -3)$

69.



As p increases, the graph of $x^2 = 4py$ gets wider.

71. $[16(4 + 3\sqrt{3} - 2\pi]/3 \approx 15.536 \text{ ft}^2$

73. Minimum distance: 147,099,713.4 km
Maximum distance: 152,096,286.6 km

75. About 0.9372 **77.** $e \approx 0.9671$

79. (a) Area $= 2\pi$ (b) Volume $= 8\pi/3$
Surface area $= [2\pi(9 + 4\sqrt{3}\pi)]/9 \approx 21.48$
(c) Volume $= 16\pi/3$

Surface area $= \dfrac{4\pi\left[6 + \sqrt{3}\ln(2 + \sqrt{3})\right]}{3} \approx 34.69$

81. 37.96 **83.** 40 **85.** $(x - 6)^2/9 - (y - 2)^2/7 = 1$

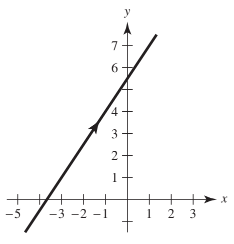
87. $x \approx 110.3$ mi **89.** Proof

91. False. See the definition of a parabola **93.** True **95.** True

97. Putnam Problem B4, 1976

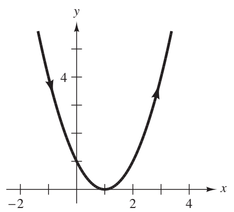
Section 10.2 *(page 703)*

1.



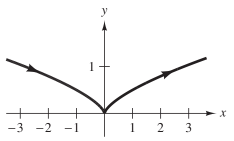
$3x - 2y + 11 = 0$

3.



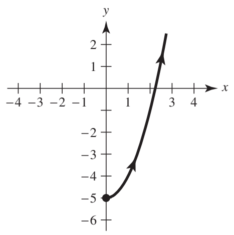
$y = (x - 1)^2$

5.



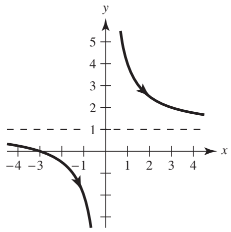
$y = \frac{1}{2}x^{2/3}$

7.



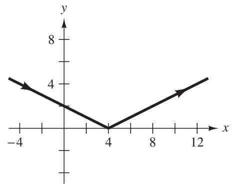
$y = x^2 - 5, \quad x \geq 0$

9.



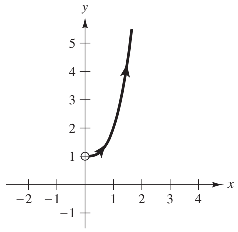
$y = (x + 3)/x$

11.



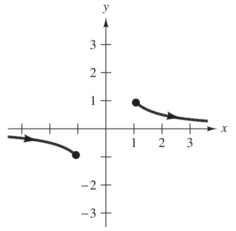
$y = |x - 4|/2$

13.



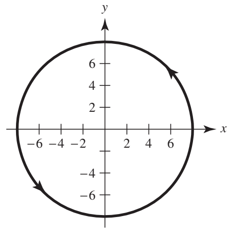
$y = x^3 + 1, \quad x > 0$

15.



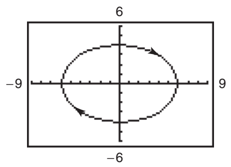
$y = 1/x, \quad |x| \geq 1$

17.



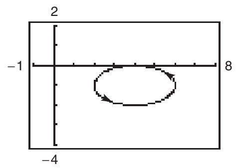
$x^2 + y^2 = 64$

19.



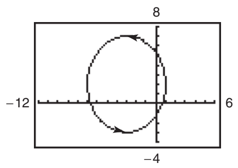
$\dfrac{x^2}{36} + \dfrac{y^2}{16} = 1$

21.



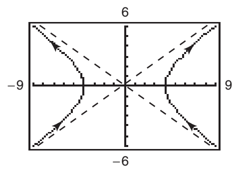
$\dfrac{(x-4)^2}{4} + \dfrac{(y+1)^2}{1} = 1$

23.



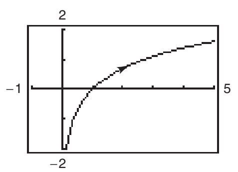
$\dfrac{(x+3)^2}{16} + \dfrac{(y-2)^2}{25} = 1$

25.



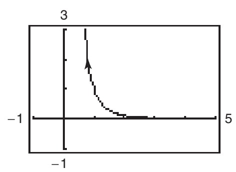
$\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

27.



$y = \ln x$

29.



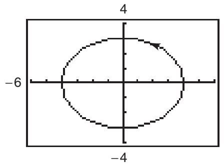
$y = \dfrac{1}{x^3}, \quad x > 0$

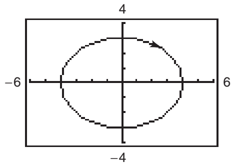
31. Each curve represents a portion of the line $y = 2x + 1$.

	Domain	Orientation	Smooth
(a)	$-\infty < x < \infty$	Up	Yes
(b)	$-1 \le x \le 1$	Oscillates	No, $\dfrac{dx}{d\theta} = \dfrac{dy}{d\theta} = 0$ when $\theta = 0, \pm\pi, \pm 2\pi, \ldots$
(c)	$0 < x < \infty$	Down	Yes
(d)	$0 < x < \infty$	Up	Yes

33. (a) and (b) represent the parabola $y = 2(1 - x^2)$ for $-1 \le x \le 1$. The curve is smooth. The orientation is from right to left in part (a) and in part (b).

35. (a)





(b) The orientation is reversed.
(c) The orientation is reversed.
(d) Answers will vary. For example,
$x = 2 \sec t$ $\quad$ $x = 2 \sec(-t)$
$y = 5 \sin t$ $\quad$ $y = 5 \sin(-t)$
have the same graphs, but their orientations are reversed.

37. $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ **39.** $\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$

41. $x = 4t$
$y = -7t$
(Solution is not unique.)

43. $x = 3 + 2 \cos \theta$
$y = 1 + 2 \sin \theta$
(Solution is not unique.)

45. $x = 10 \cos \theta$
$y = 6 \sin \theta$
(Solution is not unique.)

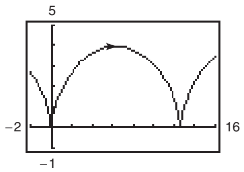
47. $x = 4 \sec \theta$
$y = 3 \tan \theta$
(Solution is not unique.)

49. $x = t$
$y = 6t - 5$;
$x = t + 1$
$y = 6t + 1$
(Solution is not unique.)

51. $x = t$
$y = t^3$;
$x = \tan t$
$y = \tan^3 t$
(Solution is not unique.)

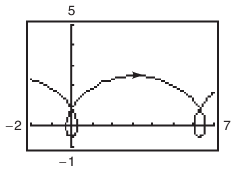
53. $x = t + 3, y = 2t + 1$ **55.** $x = t, y = t^2$

57.



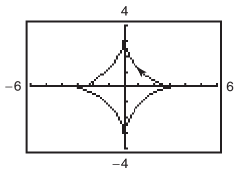
Not smooth at $\theta = 2n\pi$

59.



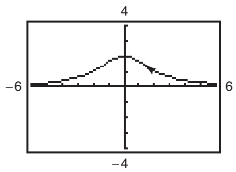
Smooth everywhere

61.



Not smooth at $\theta = \frac{1}{2}n\pi$

63.



Smooth everywhere

65. A plane curve C is a set of parametric equations, $x = f(t)$ and $y = g(t)$, and the graph of the parametric equations.

67. A curve C represented by $x = f(t)$ and $y = g(t)$ on an interval I is called smooth when f' and g' are continuous on I and not simultaneously 0, except possibly at the endpoints of I.

69. d; (4, 0) is on the graph. **71.** b; (1, 0) is on the graph.

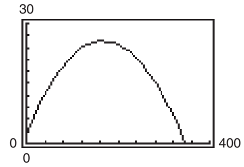
73. $x = a\theta - b \sin \theta; y = a - b \cos \theta$

75. False. The graph of the parametric equations is the portion of the line $y = x$ when $x \ge 0$.

77. True

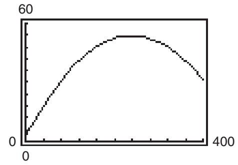
79. (a) $x = \left(\frac{440}{3} \cos \theta\right)t;\ y = 3 + \left(\frac{440}{3} \sin \theta\right)t - 16t^2$

(b)



Not a home run

(c)



Home run

(d) 19.4°

Section 10.3 *(page 711)*

1. $-3/t$ **3.** -1

5. $\dfrac{dy}{dx} = \dfrac{3}{4}, \dfrac{d^2y}{dx^2} = 0$; Neither concave upward nor concave downward

7. $dy/dx = 2t + 3,\ d^2y/dx^2 = 2$
At $t = -1, dy/dx = 1,\ d^2y/dx^2 = 2$; Concave upward

9. $dy/dx = -\cot \theta,\ d^2y/dx^2 = -(\csc \theta)^3/4$
At $\theta = \pi/4, dy/dx = -1,\ d^2y/dx^2 = -\sqrt{2}/2$;
Concave downward

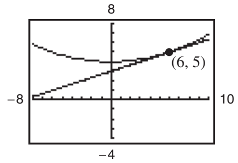
11. $dy/dx = 2 \csc \theta,\ d^2y/dx^2 = -2 \cot^3 \theta$
At $\theta = \pi/6, dy/dx = 4,\ d^2y/dx^2 = -6\sqrt{3}$;
Concave downward

13. $dy/dx = -\tan \theta,\ d^2y/dx^2 = \sec^4 \theta \csc \theta/3$
At $\theta = \pi/4, dy/dx = -1,\ d^2y/dx^2 = 4\sqrt{2}/3$;
Concave upward

15. $\left(-2/\sqrt{3}, 3/2\right)$: $3\sqrt{3}x - 8y + 18 = 0$
$(0, 2)$: $y - 2 = 0$
$\left(2\sqrt{3}, 1/2\right)$: $\sqrt{3}x + 8y - 10 = 0$

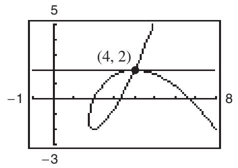
17. $(0, 0)$: $2y - x = 0$
$(-3, -1)$: $y + 1 = 0$
$(-3, 3)$: $2x - y + 9 = 0$

19. (a) and (d)



(b) At $t = 1,\ dx/dt = 6$, $dy/dt = 2$, and $dy/dx = 1/3$.
(c) $y = \frac{1}{3}x + 3$

21. (a) and (d)



(b) At $t = -1,\ dx/dt = -3$, $dy/dt = 0$, and $dy/dx = 0$.
(c) $y = 2$

23. $y = \pm\frac{3}{4}x$ **25.** $y = 3x - 5$ and $y = 1$

27. Horizontal: $(1, 0), (-1, \pi), (1, -2\pi)$
Vertical: $(\pi/2, 1), (-3\pi/2, -1), (5\pi/2, 1)$

29. Horizontal: (4, 0)
Vertical: None

31. Horizontal: $(5, -2), (3, 2)$
Vertical: None

33. Horizontal: $(0, 3), (0, -3)$
Vertical: $(3, 0), (-3, 0)$

35. Horizontal: $(5, -1), (5, -3)$
Vertical: $(8, -2), (2, -2)$

37. Horizontal: None
Vertical: $(1, 0), (-1, 0)$

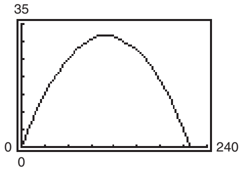
39. Concave downward: $-\infty < t < 0$
Concave upward: $0 < t < \infty$

41. Concave upward: $t > 0$

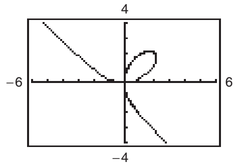
43. Concave downward: $0 < t < \pi/2$
Concave upward: $\pi/2 < t < \pi$

45. $4\sqrt{13} \approx 14.422$ **47.** $\sqrt{2}(1 - e^{-\pi/2}) \approx 1.12$

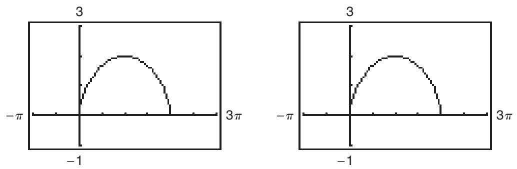
49. $\frac{1}{12}\left[\ln\left(\sqrt{37} + 6\right) + 6\sqrt{37}\right] \approx 3.249$ **51.** $6a$ **53.** $8a$

55. (a)


(b) 219.2 ft
(c) 230.8 ft

57. (a)


(b) $(0, 0), \left(4\sqrt[3]{2}/3, 4\sqrt[3]{4}/3\right)$
(c) About 6.557

59. (a)


(b) The average speed of the particle on the second path is twice the average speed of the particle on the first path.
(c) 4π

61. $S = 2\pi\int_0^4 \sqrt{10}(t + 2)\,dt = 32\pi\sqrt{10} \approx 317.907$

63. $S = 2\pi\int_0^{\pi/2} \left(\sin\theta\cos\theta\sqrt{4\cos^2\theta + 1}\right) d\theta$
$= \dfrac{\left(5\sqrt{5} - 1\right)\pi}{6}$
≈ 5.330

65. (a) $27\pi\sqrt{13}$ (b) $18\pi\sqrt{13}$ **67.** 50π **69.** $12\pi a^2/5$

71. See Theorem 10.7, Parametric Form of the Derivative, on page 706.

73. 6

75. (a) $S = 2\pi\int_a^b g(t)\sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$

(b) $S = 2\pi\int_a^b f(t)\sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$

77. Proof **79.** $3\pi/2$ **81.** d **82.** b **83.** f **84.** c

85. a **86.** e **87.** $\left(\frac{3}{4}, \frac{8}{5}\right)$ **89.** 288π

91. (a) $dy/dx = \sin\theta/(1 - \cos\theta)$; $d^2y/dx^2 = -1/[a(\cos\theta - 1)^2]$

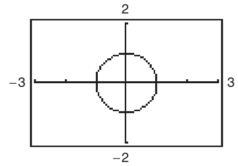
(b) $y = \left(2 + \sqrt{3}\right)\left[x - a\left(\pi/6 - \frac{1}{2}\right)\right] + a\left(1 - \sqrt{3}/2\right)$

(c) $(a(2n + 1)\pi, 2a)$

(d) Concave downward on $(0, 2\pi), (2\pi, 4\pi)$, etc.

(e) $s = 8a$

93. Proof

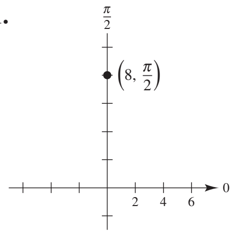
95. (a)


(b) Circle of radius 1 and center at $(0, 0)$ except the point $(-1, 0)$

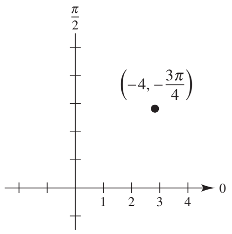
(c) As t increases from -20 to 0, the speed increases, and as t increases from 0 to 20, the speed decreases.

97. False. $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\left[\dfrac{g'(t)}{f'(t)}\right]}{f'(t)} = \dfrac{f'(t)g''(t) - g'(t)f''(t)}{[f'(t)]^3}$.

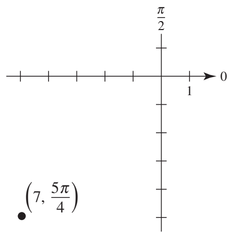
Section 10.4 *(page 722)*

1.


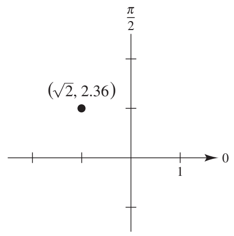
$(0, 8)$

3.


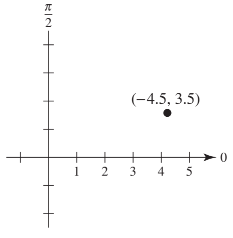
$\left(2\sqrt{2}, 2\sqrt{2}\right) \approx (2.828, 2.828)$

5.


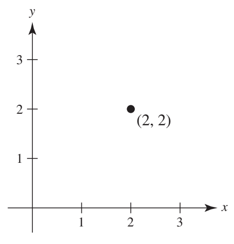
$(-4.95, -4.95)$

7.


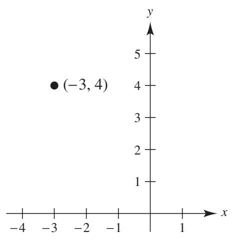
$(-1.004, 0.996)$

9.


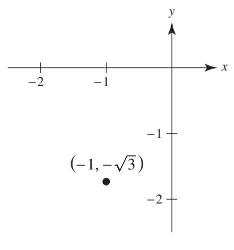
$(4.214, 1.579)$

11.


$\left(2\sqrt{2}, \pi/4\right), \left(-2\sqrt{2}, 5\pi/4\right)$

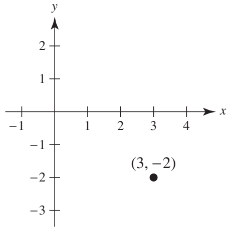
13.


$(5, 2.214), (-5, 5.356)$

15.


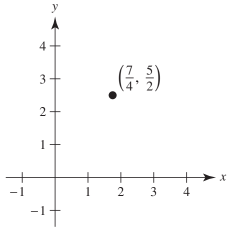
$(2, 4\pi/3), (-2, \pi/3)$

17.



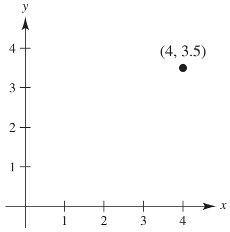
(3.606, −0.588)
(−3.606, 2.554)

19.

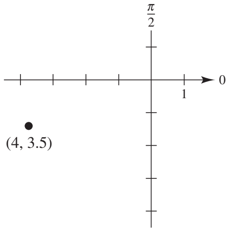


(3.052, 0.960)
(−3.052, 4.102)

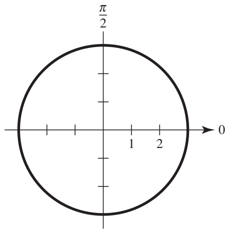
21. (a)



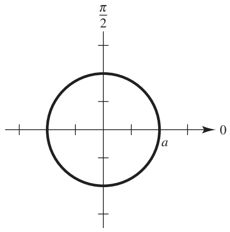
(b)



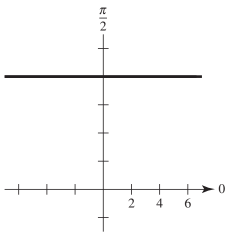
23. $r = 3$



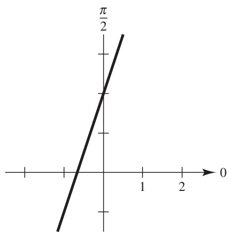
25. $r = a$



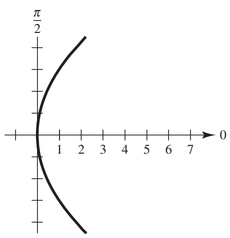
27. $r = 8 \csc \theta$



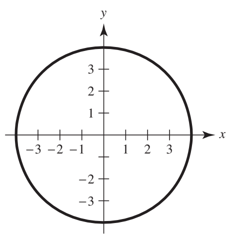
29. $r = \dfrac{-2}{3 \cos \theta - \sin \theta}$



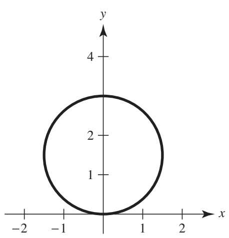
31. $r = 9 \csc^2 \theta \cos \theta$



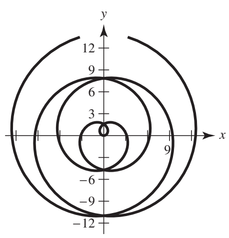
33. $x^2 + y^2 = 16$



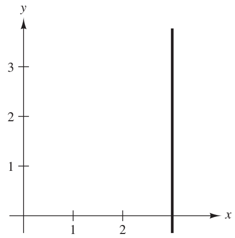
35. $x^2 + y^2 - 3y = 0$



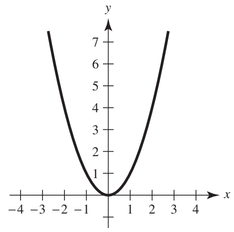
37. $\sqrt{x^2 + y^2} = \arctan(y/x)$



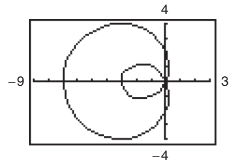
39. $x - 3 = 0$



41. $x^2 - y = 0$

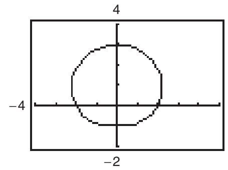


43.



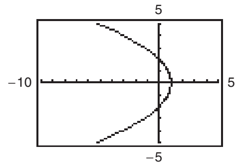
$0 \le \theta < 2\pi$

45.



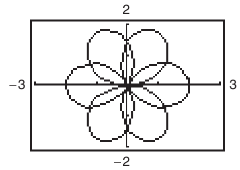
$0 \le \theta < 2\pi$

47.



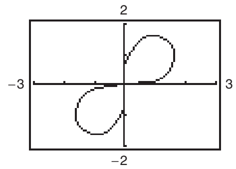
$-\pi < \theta < \pi$

49.



$0 \le \theta < 4\pi$

51.



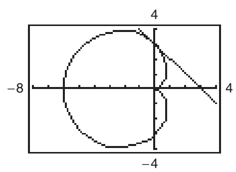
$0 \le \theta < \pi/2$

53. $(x - h)^2 + (y - k)^2 = h^2 + k^2$
Radius: $\sqrt{h^2 + k^2}$
Center: (h, k)

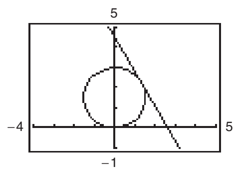
55. $\sqrt{17}$ **57.** About 5.6

59. $\dfrac{dy}{dx} = \dfrac{2 \cos \theta(3 \sin \theta + 1)}{6 \cos^2 \theta - 2 \sin \theta - 3}$
$(5, \pi/2)$: $dy/dx = 0$
$(2, \pi)$: $dy/dx = -2/3$
$(-1, 3\pi/2)$: $dy/dx = 0$

61. (a) and (b) (c) $dy/dx = -1$



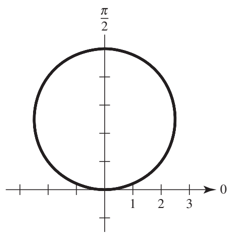
63. (a) and (b) (c) $dy/dx = -\sqrt{3}$



65. Horizontal: $(2, 3\pi/2), \left(\frac{1}{2}, \pi/6\right), \left(\frac{1}{2}, 5\pi/6\right)$
Vertical: $\left(\frac{3}{2}, 7\pi/6\right), \left(\frac{3}{2}, 11\pi/6\right)$

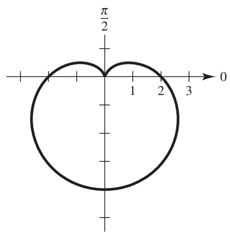
67. $(5, \pi/2), (1, 3\pi/2)$

69.



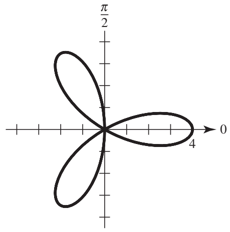
$\theta = 0$

71.



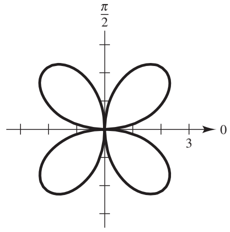
$\theta = \pi/2$

73.



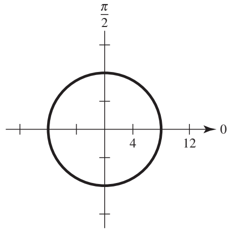
$\theta = \pi/6, \pi/2, 5\pi/6$

75.

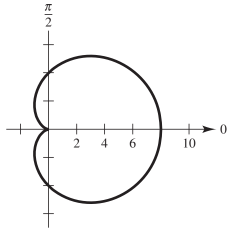


$\theta = 0, \pi/2$

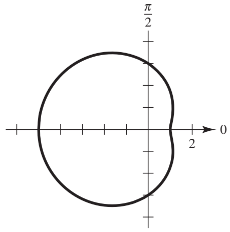
77.



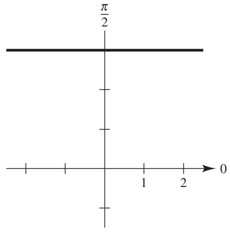
79.



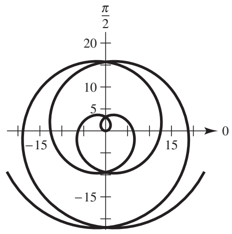
81.



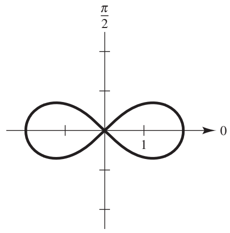
83.



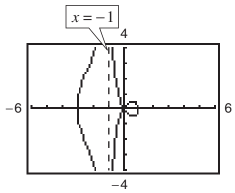
85.



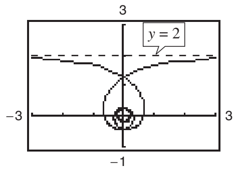
87.



89.



91.



93. The rectangular coordinate system is a collection of points of the form (x, y), where x is the directed distance from the y-axis to the point and y is the directed distance from the x-axis to the point. Every point has a unique representation.

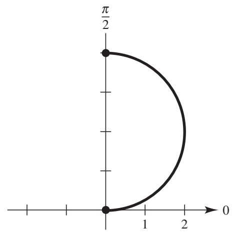
The polar coordinate system is a collection of points of the form (r, θ), where r is the directed distance from the origin O to a point P and θ is the directed angle, measured counterclockwise, from the polar axis to the segment $\overline{OP}$. Polar coordinates do not have unique representations.

95. Slope of tangent line to graph of $r = f(\theta)$ at (r, θ) is

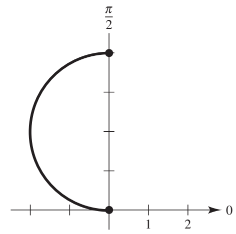
$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}.$$

If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then $\theta = \alpha$ is tangent at the pole.

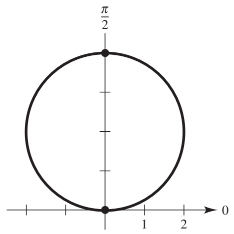
97. (a)



(b)



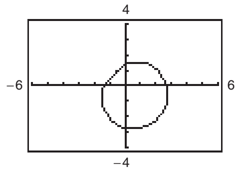
(c)



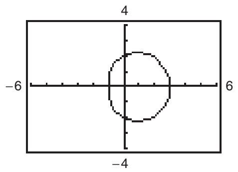
99. Proof

101. (a) $r = 2 - \sin(\theta - \pi/4)$

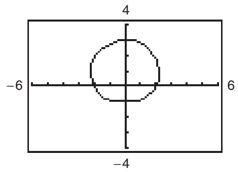
$$= 2 - \frac{\sqrt{2}(\sin\theta - \cos\theta)}{2}$$



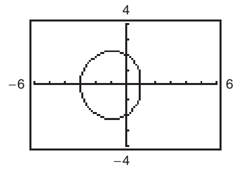
(b) $r = 2 + \cos\theta$



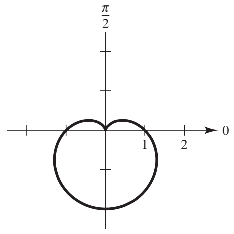
(c) $r = 2 + \sin\theta$



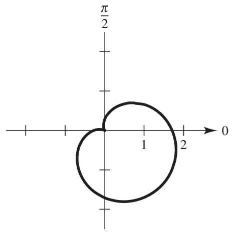
(d) $r = 2 - \cos\theta$



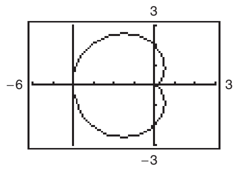
103. (a)



(b)

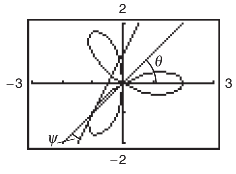


105.

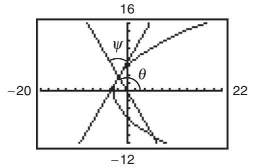


$\psi = \pi/2$

107.



$\psi = \arctan\frac{1}{3} \approx 18.4°$

109.


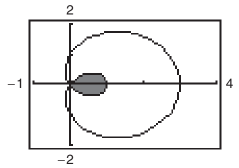
$\psi = \pi/3$, 60°

111. True **113.** True

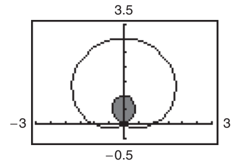
Section 10.5 *(page 731)*

1. $8\int_0^{\pi/2} \sin^2 \theta \, d\theta$ **3.** $\frac{1}{2}\int_{\pi/2}^{3\pi/2} (3 - 2\sin\theta)^2 \, d\theta$ **5.** 9π

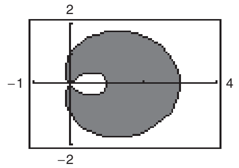
7. $\pi/3$ **9.** $\pi/8$ **11.** $3\pi/2$ **13.** 27π **15.** 4

17.


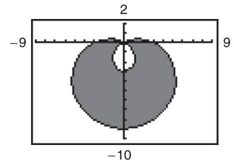
$(2\pi - 3\sqrt{3})/2$

19.


$(2\pi - 3\sqrt{3})/2$

21.


$\pi + 3\sqrt{3}$

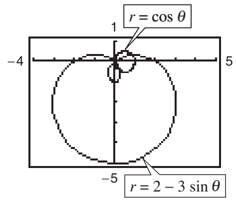
23.


$9\pi + 27\sqrt{3}$

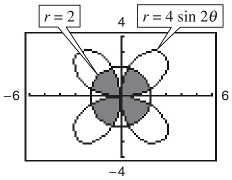
25. $(1, \pi/2), (1, 3\pi/2), (0, 0)$

27. $\left(\frac{2-\sqrt{2}}{2}, \frac{3\pi}{4}\right), \left(\frac{2+\sqrt{2}}{2}, \frac{7\pi}{4}\right), (0, 0)$

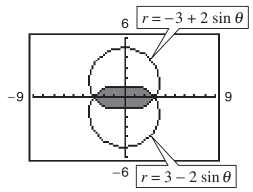
29. $\left(\frac{3}{2}, \frac{\pi}{6}\right), \left(\frac{3}{2}, \frac{5\pi}{6}\right), (0, 0)$ **31.** $(2, 4), (-2, -4)$

33.


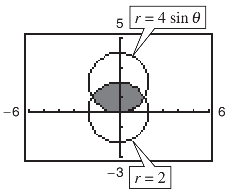
(0, 0), (0.935, 0.363),
(0.535, −1.006)
The graphs reach the pole at different times (θ-values).

35.


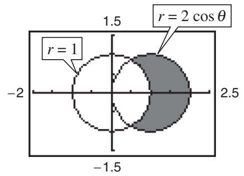
$\frac{4}{3}(4\pi - 3\sqrt{3})$

37.


$11\pi - 24$

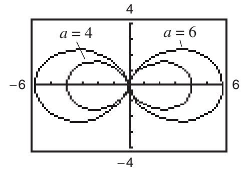
39.


$\frac{2}{3}(4\pi - 3\sqrt{3})$

41.


$\pi/3 + \sqrt{3}/2$

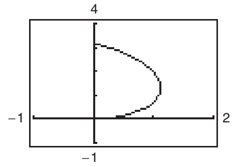
43. $5\pi a^2/4$ **45.** $(a^2/2)(\pi - 2)$

47. (a) $(x^2 + y^2)^{3/2} = ax^2$
(b)


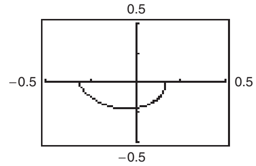
(c) $15\pi/2$

49. The area enclosed by the function is $\pi a^2/4$ if n is odd and is $\pi a^2/2$ if n is even.

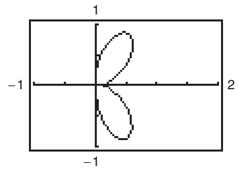
51. 16π **53.** 4π **55.** 8

57.


About 4.16

59.


About 0.71

61.


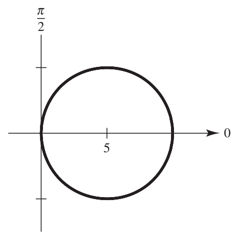
About 4.39

63. 36π **65.** $\frac{2\pi\sqrt{1+a^2}}{1+4a^2}(e^{\pi a} - 2a)$ **67.** 21.87

69. You will only find simultaneous points of intersection. There may be intersection points that do not occur with the same coordinates in the two graphs.

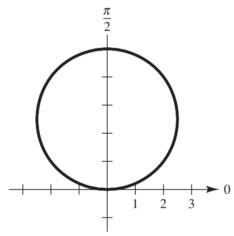
71. (a) Circle of radius 5

Area $= 25\pi$



(b) Circle of radius $5/2$

Area $= \frac{25}{4}\pi$



73. $40\pi^2$

75. (a) 16π
(b)

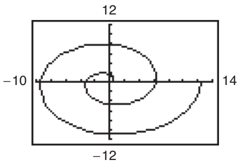
θ	0.2	0.4	0.6	0.8	1.0	1.2	1.4
A	6.32	12.14	17.06	20.80	23.27	24.60	25.08

(c) and (d) For $\frac{1}{4}$ of area ($4\pi \approx 12.57$): 0.42
For $\frac{1}{2}$ of area ($8\pi \approx 25.13$): 1.57($\pi/2$)
For $\frac{3}{4}$ of area ($12\pi \approx 37.70$): 2.73

(e) No. The results do not depend on the radius. Answers will vary.

77. Circle

79. (a)



The graph becomes larger and more spread out. The graph is reflected over the y-axis.

(b) $(an\pi, n\pi)$, where $n = 1, 2, 3, \dots$

(c) About 21.26 (d) $4/3\pi^3$

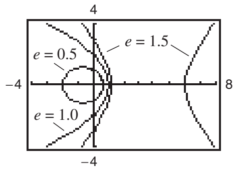
81. $r = \sqrt{2}\cos\theta$

83. False. The graphs of $f(\theta) = 1$ and $g(\theta) = -1$ coincide.

85. Proof

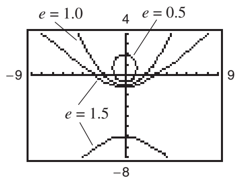
Section 10.6 *(page 739)*

1.



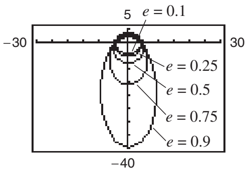
(a) Parabola
(b) Ellipse
(c) Hyperbola

3.



(a) Parabola
(b) Ellipse
(c) Hyperbola

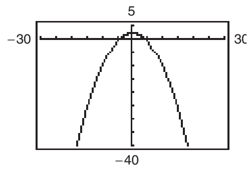
5. (a)



Ellipse

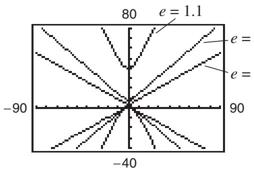
As $e \to 1^-$, the ellipse becomes more elliptical, and as $e \to 0^+$, it becomes more circular.

(b)



Parabola

(c)

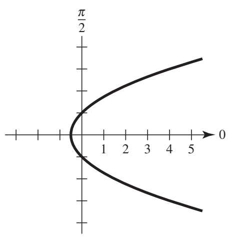


Hyperbola

As $e \to 1^+$, the hyperbola opens more slowly, and as $e \to \infty$, it opens more rapidly.

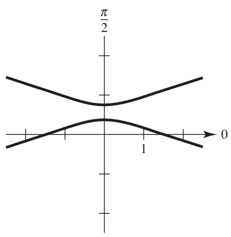
7. c **8.** f **9.** a **10.** e **11.** b **12.** d

13. $e = 1$
Distance $= 1$



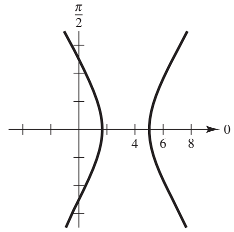
Parabola

15. $e = 3$
Distance $= \frac{1}{2}$



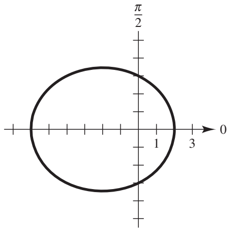
Hyperbola

17. $e = 2$
Distance $= \frac{5}{2}$



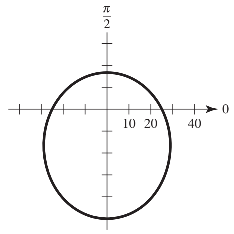
Hyperbola

19. $e = \frac{1}{2}$
Distance $= 6$



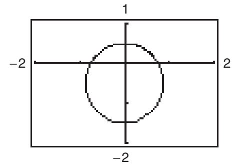
Ellipse

21. $e = \frac{1}{2}$
Distance $= 50$



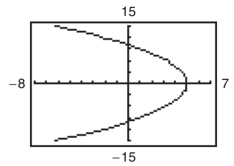
Ellipse

23.



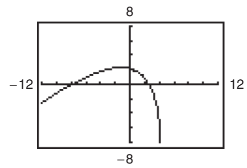
Ellipse
$e = \frac{1}{2}$

25.



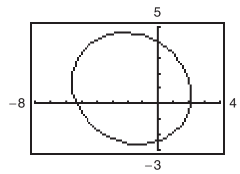
Parabola
$e = 1$

27.



Rotated $\pi/3$ radian counterclockwise.

29.



Rotated $\pi/6$ radian clockwise.

31. $r = \dfrac{8}{8 + 5\cos\left(\theta + \dfrac{\pi}{6}\right)}$

33. $r = 3/(1 - \cos\theta)$ **35.** $r = 1/(2 + \sin\theta)$

37. $r = 2/(1 + 2\cos\theta)$ **39.** $r = 2/(1 - \sin\theta)$

41. $r = 16/(5 + 3\cos\theta)$ **43.** $r = 9/(4 - 5\sin\theta)$

45. $r = 4/(2 + \cos\theta)$

47. If $0 < e < 1$, the conic is an ellipse.
If $e = 1$, the conic is a parabola.
If $e > 1$, the conic is a hyperbola.

49. If the foci are fixed and $e \to 0$, then $d \to \infty$. To see this, compare the ellipses

$$r = \frac{1/2}{1 + (1/2)\cos\theta},\ e = \frac{1}{2},\ d = 1 \text{ and}$$

$$r = \frac{5/16}{1 + (1/4)\cos\theta},\ e = \frac{1}{4},\ d = \frac{5}{4}.$$

51. Proof

53. $r^2 = \dfrac{9}{1 - (16/25)\cos^2\theta}$ **55.** $r^2 = \dfrac{-16}{1 - (25/9)\cos^2\theta}$

57. About 10.88 **59.** 3.37

61. $\dfrac{7979.21}{1 - 0.9372\cos\theta}$; 11,015 mi

63. $r = \dfrac{149{,}558{,}278.0560}{1 - 0.0167\cos\theta}$
Perihelion: 147,101,680 km
Aphelion: 152,098,320 km

65. $r = \dfrac{4{,}497{,}667{,}328}{1 - 0.0086\cos\theta}$
Perihelion: 4,459,317,200 km
Aphelion: 4,536,682,800 km

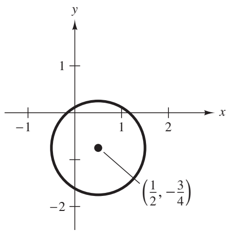
67. Answers will vary. Sample answers:
(a) 3.591×10^{18} km^2; 9.322 yr
(b) $\alpha \approx 0.361 + \pi$; Larger angle with the smaller ray to generate an equal area
(c) Part (a): 1.583×10^9 km; 1.698×10^8 km/yr
Part (b): 1.610×10^9 km; 1.727×10^8 km/yr

69. Proof

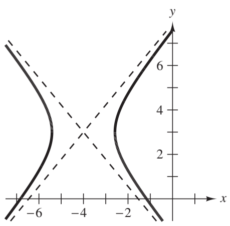
Review Exercises for Chapter 10 *(page 742)*

1. e **2.** c **3.** b **4.** d **5.** a **6.** f

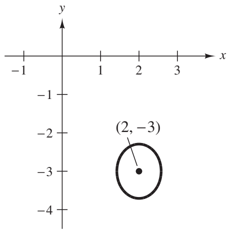
7. Circle
Center: $\left(\frac{1}{2}, -\frac{3}{4}\right)$
Radius: 1



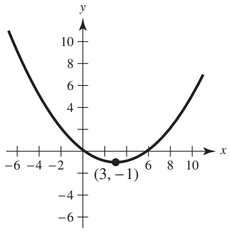
9. Hyperbola
Center: $(-4, 3)$
Vertices: $(-4 \pm \sqrt{2}, 3)$
Foci: $(-4 \pm \sqrt{5}, 3)$
$e = \sqrt{\dfrac{5}{2}}$
Asymptotes:
$y = 3 + \dfrac{\sqrt{3}}{\sqrt{2}}(x + 4)$;
$y = 3 - \dfrac{\sqrt{3}}{\sqrt{2}}(x + 4)$



11. Ellipse
Center: $(2, -3)$
Vertices: $(2, -3 \pm \sqrt{2}/2)$
$e = \sqrt{\dfrac{1}{3}}$



13. Parabola
Vertex: $(3, -1)$
Focus: $(3, 1)$
Directrix: $y = -3$
$e = 1$



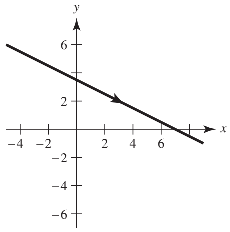
15. $y^2 - 4y - 12x + 4 = 0$

17. $\dfrac{x^2}{49} + \dfrac{y^2}{24} = 1$ **19.** $\dfrac{(x-3)^2}{5} + \dfrac{(y-4)^2}{9} = 1$

21. $\dfrac{y^2}{64} - \dfrac{x^2}{16} = 1$ **23.** $\dfrac{x^2}{49} - \dfrac{(y+1)^2}{32} = 1$

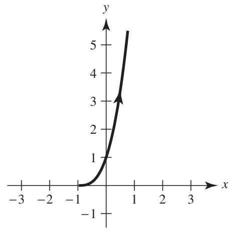
25. (a) $(0, 50)$ (b) About 38,294.49

27.



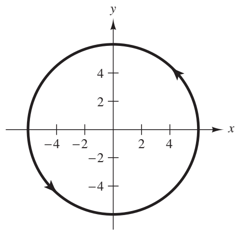
$x + 2y - 7 = 0$

29.



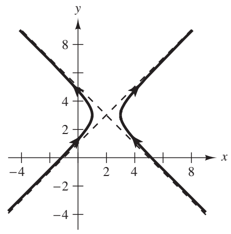
$y = (x + 1)^3, x > -1$

31.



$x^2 + y^2 = 36$

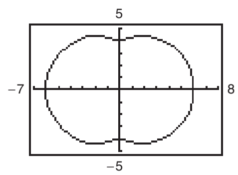
33.



$(x - 2)^2 - (y - 3)^2 = 1$

35. $x = t, y = 4t + 3; x = t + 1, y = 4t + 7$
(Solution is not unique.)

37.



39. $\dfrac{dy}{dx} = -\dfrac{4}{5}, \dfrac{d^2y}{dx^2} = 0$
At $t = 3$, $\dfrac{dy}{dx} = -\dfrac{4}{5}, \dfrac{d^2y}{dx^2} = 0$; Neither concave upward or concave downward

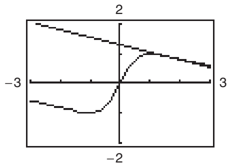
41. $\dfrac{dy}{dx} = -2t^2, \dfrac{d^2y}{dx^2} = 4t^3$
At $t = -1$, $\dfrac{dy}{dx} = -2, \dfrac{d^2y}{dx^2} = -4$; Concave downward

43. $\dfrac{dy}{dx} = -4\cot\theta, \dfrac{d^2y}{dx^2} = -4\csc^3\theta$
At $\theta = \dfrac{\pi}{6}, \dfrac{dy}{dx} = -4\sqrt{3}, \dfrac{d^2y}{dx^2} = -32$; Concave downward

45. $\frac{dy}{dx} = -4\tan\theta,\ \frac{d^2y}{dx^2} = \frac{4}{3}\sec^4\theta\csc\theta$

At $\theta = \frac{\pi}{3},\ \frac{dy}{dx} = -4\sqrt{3},\ \frac{d^2y}{dx^2} = \frac{128\sqrt{3}}{9}$; Concave upward

47. (a) and (d)



(b) $dx/d\theta = -4,\ dy/d\theta = 1,\ dy/dx = -\frac{1}{4}$

(c) $y = -\frac{1}{4}x + \frac{3\sqrt{3}}{4}$

49. Horizontal: $(5, 0)$
Vertical: None

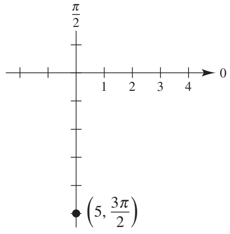
51. Horizontal: $(2, 2), (2, 0)$
Vertical: $(4, 1), (0, 1)$

53. $\frac{1}{54}(145^{3/2} - 1) \approx 32.315$

55. (a) $s = 12\pi\sqrt{10} \approx 119.215$
(b) $s = 4\pi\sqrt{10} \approx 39.738$

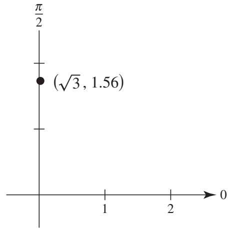
57. $A = 3\pi$

59.



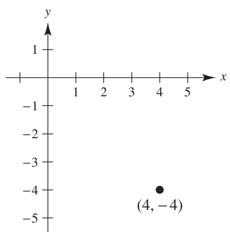
Rectangular: $(0, -5)$

61.



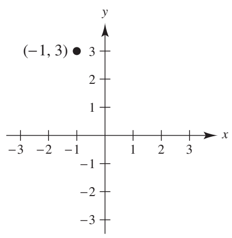
Rectangular: $(0.0187, 1.7320)$

63.



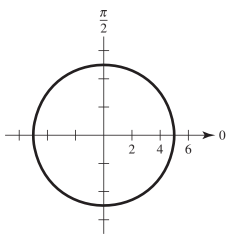
$\left(4\sqrt{2}, \frac{7\pi}{4}\right), \left(-4\sqrt{2}, \frac{3\pi}{4}\right)$

65.

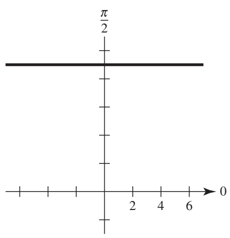


$\left(\sqrt{10}, 1.89\right), \left(-\sqrt{10}, 5.03\right)$

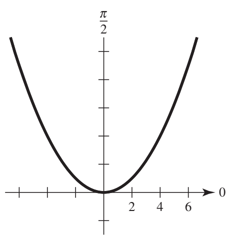
67. $r = 5$



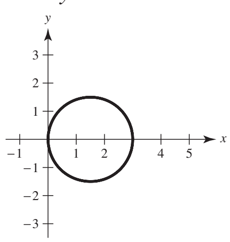
69. $r = 9\csc\theta$



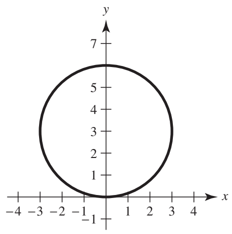
71. $r = 4\tan\theta\sec\theta$



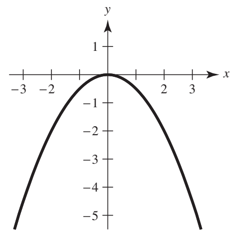
73. $x^2 + y^2 - 3x = 0$



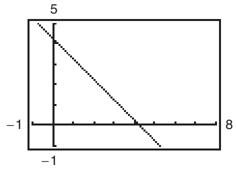
75. $x^2 + (y - 3)^2 = 9$



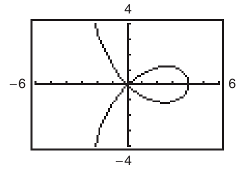
77. $y = -\frac{1}{2}x^2$



79.



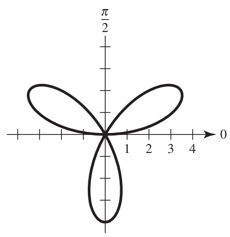
81.



83. Horizontal: $\left(\frac{3}{2}, \frac{2\pi}{3}\right), \left(\frac{3}{2}, \frac{4\pi}{3}\right)$

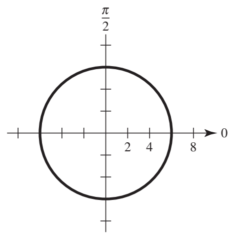
Vertical: $\left(\frac{1}{2}, \frac{\pi}{3}\right), (2, \pi), \left(\frac{1}{2}, \frac{5\pi}{3}\right)$

85.

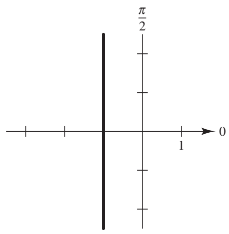


$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$

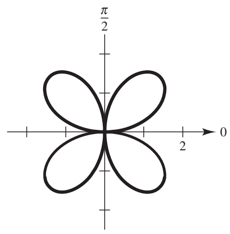
87. Circle



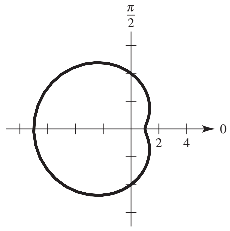
89. Line



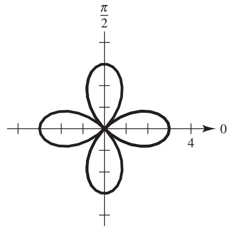
91. Rose curve



93. Limaçon

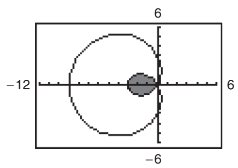


95. Rose curve



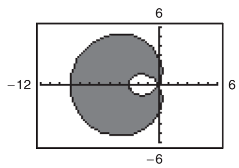
97. $\frac{9\pi}{20}$ **99.** $\frac{9\pi}{2}$ **101.** 4

103.



$9\pi - \frac{27\sqrt{3}}{2}$

105.

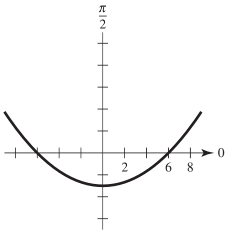


$9\pi + 27\sqrt{3}$

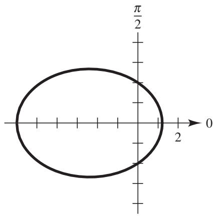
107. $\left(1 + \frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right), \left(1 - \frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right), (0, 0)$ **109.** $\frac{5\pi}{2}$

111. $S = 2\pi \int_0^{\pi/2} (1 + 4\cos\theta)\sin\theta\sqrt{17 + 8\cos\theta}\,d\theta$
$= 34\pi\sqrt{17}/5 \approx 88.08$

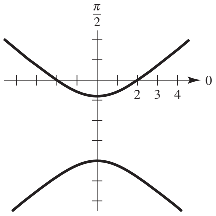
113. Parabola
$e = 1$; Distance $= 6$;



115. Ellipse
$e = \frac{2}{3}$; Distance $= 3$;



117. Hyperbola
$e = \frac{3}{2}$; Distance $= \frac{4}{3}$;

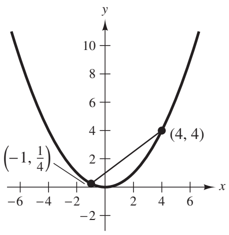


119. $r = \dfrac{4}{1 + \cos\theta}$ **121.** $r = \dfrac{9}{1 + 3\sin\theta}$

123. $r = \dfrac{5}{3 - 2\cos\theta}$

P.S. Problem Solving *(page 745)*

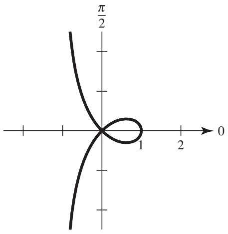
1. (a)



(b) and (c) Proofs

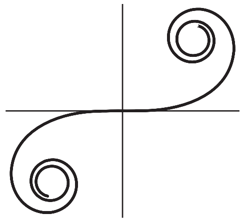
3. Proof

5. (a) $y^2 = x^2[(1 - x)/(1 + x)]$
(b) $r = \cos 2\theta \cdot \sec\theta$
(c)



(d) $y = x, y = -x$
(e) $\left(\frac{\sqrt{5} - 1}{2}, \pm\frac{\sqrt{5} - 1}{2}\sqrt{-2 + \sqrt{5}}\right)$

7. (a)



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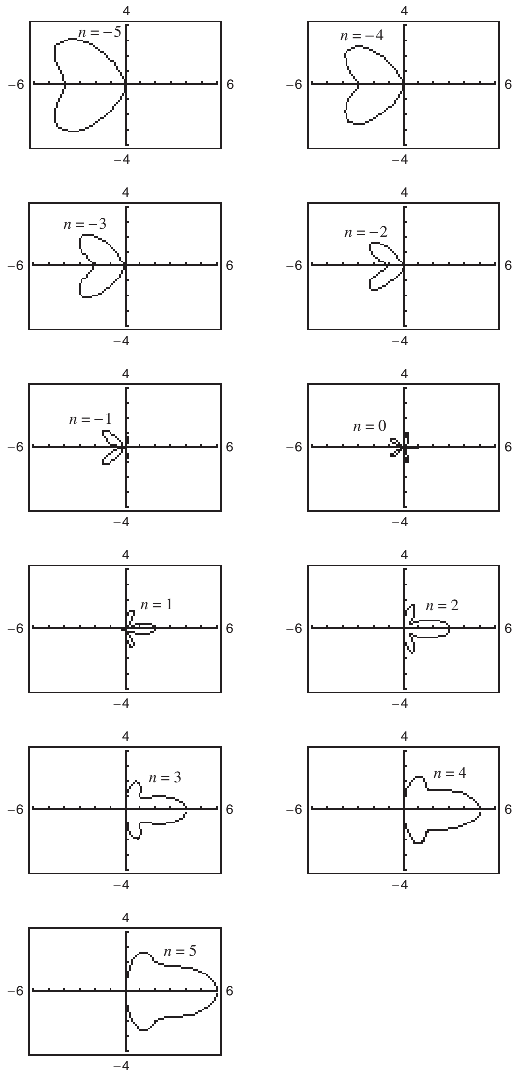
(b) Proof
(c) $a, 2\pi$

9. $A = \frac{1}{2}ab$ **11.** $r^2 = 2\cos 2\theta$

13. $r = \dfrac{d}{\sqrt{2}} e^{((\pi/4) - \theta)}, \theta \geq \dfrac{\pi}{4}$

15. (a) $r = 2a\tan\theta\sin\theta$
(b) $x = 2at^2/(1 + t^2)$
$y = 2at^3/(1 + t^2)$
(c) $y^2 = x^3/(2a - x)$

17.



$n = 1, 2, 3, 4, 5$ produce "bells"; $n = -1, -2, -3, -4, -5$ produce "hearts."